MINIPROJECT: RULED STRIP SURFACES

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A ruled surface is a surface swept out by a family of straight lines. They are used a lot in architectural construction (Figure 1) because they are easy to make. In this project you will learn first about the mathematics of ruled surfaces, and then figure out how to construct more general surfaces out of tangent continuous ruled strips.

![Figure 1. Examples of ruled surfaces in architecture.](image)

**Part 1: Introduction to Ruled Surfaces.**

1. Explain the definition of a ruled surface and some of its basic properties, including the Gaussian curvature, and condition for the surface to be “developable”. A special case is a tangent developable: explain what this is, and at which points such a surface is regular. You may consult any source for this, e.g. A. Pressley “Elementary Differential Geometry”, K. Tapp Differential Geometry of Curves and Surfaces, and a more detailed account in M. do Carmo “Differential geometry of curves and surfaces”.


**Part 2: Tangent Continuous Ruled Strips.** Ruled surfaces are somewhat limited in the shapes they are able to produce. More interesting shapes can be made by fitting together ruled strips in such a way that the tangent planes match at the joins, as in Figure 2.

1. Define a ruled strip to be a parameterized surface \( \sigma : [a,b] \times [c,d] \rightarrow \mathbb{R}^3 \) given as

   \[
   \sigma(u,v) = \gamma(u) + v\delta(u), \quad u \in [a,b], \quad v \in [c,d].
   \]

   where \( \gamma : [a,b] \rightarrow \mathbb{R}^3 \) is a regular curve and \( \delta : [a,b] \rightarrow \mathbb{R}^3 \) is a vector field never parallel to \( \gamma'(u) \). (i.e. \( \sigma \) is just a parameterized ruled surface, with a rectangular parameter space \([a,b] \times [c,d]\)).

   Suppose given a ruled strip \( \sigma : [a,b] \times [0,c_1] \rightarrow \mathbb{R}^3 \), and assume that it is everywhere regular. Show that it is possible to construct another ruled strip \( \sigma_2 \) parameterized on
Figure 2. Examples of surfaces constructed from tangent continuous ruled strips. 

\[ [a, b] \times [c_1, c_2] \] such that \( \sigma \) and \( \sigma_2 \) agree along the curve \( [a, b] \times \{c_1\} \), and where both strips have the same tangent plane along this curve, but the ruling directions are not the same for the two strips. That is, \( \sigma_2 \) is not just an extension of the rulings of the first strip. (Examples of surfaces constructed from multiple such strips are shown in Figure 2). Explain how much freedom you have in the choice of the vector field \( \delta_2 \) that defines the second ruled strip.

(2) Write a program (e.g. in Maple, Matlab, Python etc) that allows the user to construct a surface consisting of multiple tangent continuous ruled strips. If time permits, you could use your program to make realistic designs for architecture or furniture.