Weekplan: Introduction to Data Structures

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Reading

Introduction to Algorithms, Cormen, Rivest, Leisersons and Stein (CLRS): Introduction to Part III + Chapter 10.

Exercises

1 Stacks and Queues

1.1 CLRS [w] 10.1-1.

1.2 Exercise 5.1 in the exam set from 2011.

1.3 CLRS 10.1-2.

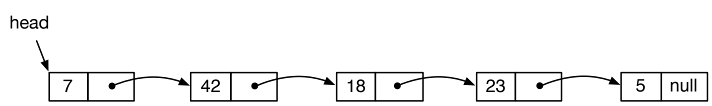
1.4 CLRS [w] 10.1-3.

1.5 CLRS 10.1-6.

2 Algorithms on Linked Lists Look at the algorithms FOO and BAR and the linked list below. Solve the following exercises.

```
FOO(head)
x = head
c = 0
while x ≠ null do
    x = x.next
    c = c + 1
end while
return c
```

```
BAR(x, s)
if x == null then
    return s
else
    return BAR(x.next, s + x.key)
end if
```



2.1 [w] Run FOO(*head*) by hand.

2.2 [w] Explain what FOO computes.

2.3 Run BAR(*head*, 0) by hand.

2.4 Explain what BAR does.

3 Implementation of Linked Lists Assume `x` is an element in a singly linked list as described in the lecture. Solve the following exercises.

3.1 [w] Assume `x` is not the last element in the list. What is the result of the following code snippet?

```
x.next = x.next.next;
```

3.2 [w] Let `t` be a new element that is not already in the list. What is the result of the following code snippet?

```
t.next = x.next;
x.next = t;
```

3.3 [w] Why does the following code snippet *not* do the same as the above?

```
x.next = t;
t.next = x.next;
```

4 Implementation of Stacks and Queues Solve the following exercises.

4.1 [†] Implement a stack that can contain integers using a singly linked list.

4.2 [†] Implement a queue that can contain integers using a singly linked list.

5 Sorted Linked Lists Let L be a singly linked list consisting of n integers in sorted order. Solve the following exercises.

5.1 Give an algorithm to insert a new integer in L such that the list is still sorted afterwards.

5.2 Professor Gørtz suggests one can improve the insertion algorithm by using binary search. Is she right?

6 List Reversal Give an algorithm to reverse a singly linked list, ie. produces a singly linked list with the elements in the reversed order. Your algorithm should run in $\Theta(n)$ time and not use more than constant extra space (besides from the space for the list itself).

7 Dynamic Arrays and Stacks We are interested in implementing a stack using a dynamic array without a maximum size for the array in the beginning. Solve the following exercises.

7.1 [$*$] Generalize dynamic arrays to also support stacks that shrinks (ie. supports both PUSH and POP operations). The running time of any sequence of n operations must be $\Theta(n)$ and at any point in time your solution should use linear time in the number of elements currently in the stack.

7.2 [$**$] Show how one can obtain $O(1)$ time per stack operation using dynamic arrays and linear space in the number of elements currently in the stack. Only consider growing stacks and thus ignore POP. *Hint:* Consider how the work can be evenly distributed over all operations.

M Mandatory Exercise: Emperical Analysis Solve the following exercises. The focus of these exercises is on the analysis part and not the implementation part. Therefore your code should just be added in an appendix.

M.1 Implement Insertion-Sort in your favorite imperative programming language.

M.2 Make a small program that generates random sequences of integers of increasing sizes, and runs your sorting algorithm on these while measuring the running time. Make sure the longest sequences take at least a some seconds to sort.

M.3 Show the results of your tests in tabular and graph form (time as a function of size). Compare the results with your expectations. Does it match with the theoretical asymptotic running time?

M.4 Repeat step 2 and 3 but this time use the standard built-in sorting method in your chosen programming language (for instance Arrays.sort in Java or sort in C/C++). Finally compare the results with the results from your own implementation.