Weekplan: Introduction to Graphs

The 02105+02326 DTU Algorithms Team

Reading

Introduction to Algorithms, Cormen, Rivest, Leisersons and Stein (CLRS): Introduction to Part VI + Chapter 22.1-22.4 + Appendix B.4-B.5.

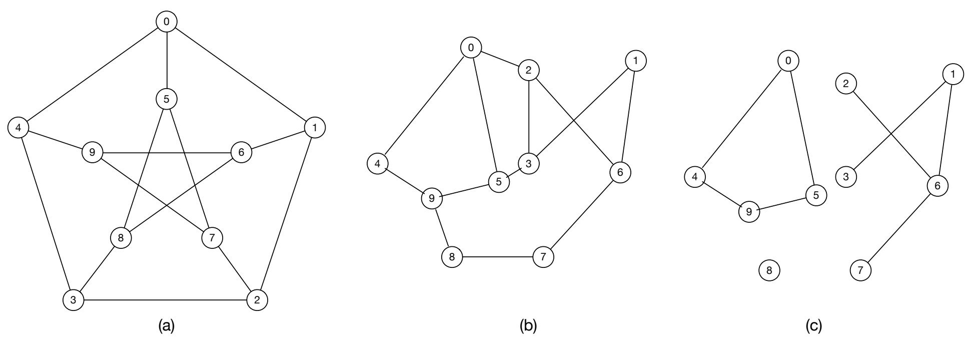


Figure 1: Graphs for the exercises. (a) is the *Petersen graph*.

Exercises

1 Representation, Properties and Algorithms Look at the graphs in Figure 1. Solve the following exercises.

- **1.1** [w] Show adjacency lists and adjacency matrices for (a) and (c).
- **1.2** [w] Simulate DFS on (a) starting in node 0. Assume the adjacency lists are sorted. Specify the DFS-tree, and start and end times.
- **1.3** [w] Simulate BFS on (a) starting in node 0. Assume the adjacency lists are sorted. Specify the BFS-tree, and the distance for each node.
- **1.4** Specify the connected components of (a), (b), and (c).
- **1.5** Which of (a), (b), and (c) are bipartite?

2 Depth First Search using a Stack Explain how to implement DFS without using recursion. *Hint:* use an (explicit) stack.

3 Find a Cycle Give an algorithm that determines if a graph i *cyclic*, ie. contains a cycle. How fast is your algorithm?

4 Labyrinths Solve exercise 3 in the exam set from 2010 (this exercise is the same in 02326 and 02105).

5 Number of Shortest Paths Give an algorithm that given two nodes s and t in G returns the *number* of shortest paths between s and t in G.

6 Implementation of Graphs We want to support the following operations on a dynamic graph G.

- ADDEDGE(u, v): add an edge between the nodes u and v.
- ADJACENT(u, v): return if u and v are adjacent in G.
- NEIGHBOURS(v): prints all neighbors of node v.

Solve the following exercises.

6.1 [†] Implement the operations on an adjacency matrix.

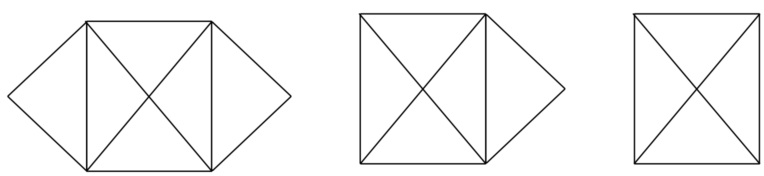
6.2 [†] Implement the operations on an adjacency list.

7 Euler Tours and Euler Paths Let G be a connected graph with n nodes and m edges. An *Euler tour* in G is a cycle that contains all edges in G exactly once. An *Euler path* in G is a path that contains all edges in G exactly once. Solve the following exercises.

7.1 [$*$] Show that G has an Euler tour if and only if all nodes have even degree.

7.2 [$*$] Show that G has an Euler path if and only if at most two nodes have have odd degree.

7.3 Which of the drawings below can you draw without lifting the pencil? Can you start and end at the same place?



7.4 Give an $O(n+m)$ time algorithm that determines if G has an Euler tour.

7.5 [$*$] Give an $O(n+m)$ algorithm that finds an Euler tour in G if it exists.

8 Diameter of Trees Let T be a tree with n nodes. The *diameter* of T is the longest shortest path between a pair of nodes in T. Solve the following exercises.

8.1 Give algorithm to compute the diameter of T in $O(n^2)$ time.

8.2 [$**$] Give algorithm to compute the diameter of T in $O(n)$ time.