Weekplan: Shortest Paths

The 02105+02326 DTU Algorithms Team

Reading

Introduction to Algorithms, Cormen, Rivest, Leisersons and Stein (CLRS): Chapter 24 excluding 24.1 and 24.4.

Exercises

1 Algorithms and Properties

1.1 [w] CLRS 24.3-1 (also show the contents of the priority queue).

1.2 [w] CLRS 24.2-1

1.3 CLRS 24.3-2

1.4 CLRS 24.3-4

1.5 Let T be a shortest path tree from a node s in a graph G. Assume we add a constant c to all edge weights in G. Is T still a shortest path tree?

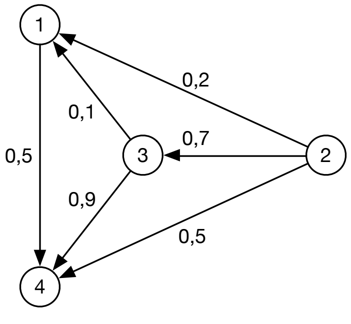
2 Cable Routing Exercise 3 from the 2012 exam set (respectively 02326 and 02105)

3 Longest Paths in DAGs Give an algorithm to find the *longest path* in a DAG.

4 [BSc] **Negative Edges** Explain where in the proof of Dijkstras algorithm we use that edge weights may not be negative.

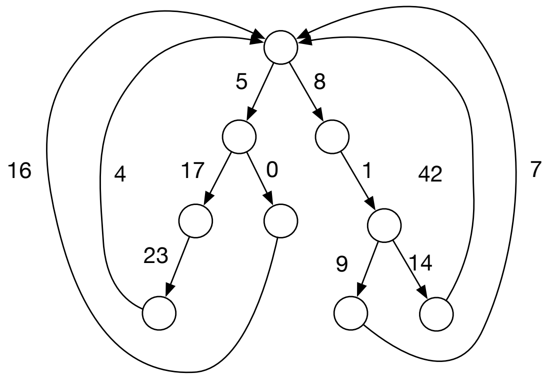
5 Node Weighted Dijkstra Let G be a directed graph where all *nodes* are associated with a non-negative weight. The weight of a path in G is the sum of the weights of the nodes on the path. Give an algorithm to compute the shortest path between two nodes in G.

6 [$*$] **Zombie Travel** In the post-apocalyptic zombie world you need to know the safest travel between two cities such that you hopefully avoid being eaten by the zombies. You are given a graph G where each node represents a city and each edge a road between two citites. Each edge e has a *probability* $s(e)$, $0 \leq s(e) \leq 1$ for surviving traveling on that edge without being eaten. The probabilities on each edge are independent and the probability of surving the entire travel along a path P is the product of the probabilities of surviving on each edge of P.



As an example look at the above graph. If you travel directly from node 2 to 4 you have 50% chance of surviving. If you instead travel via node 3 you have $0.7 \cdot 0.9 = 63\%$ chance of surviving. If you travel via 3 and 1 you only have $0.7 \cdot 0.1 \cdot 0.5 = 3.5\%$ chance of surviving. Give an algorithm that computes the safest way from a node s to another node t.

7 Loopy Trees A *loopy tree* is a weighted directed graph constructed from a binary tree by adding an edge from each leaf to the root. All edges have non-negative weights.



7.1 How long time does Dijkstras algorithm use to compute the shortest path from a node s in a loopy tree?

7.2 [$*$] Give a faster algorithm.