Weekplan: Binary Search Trees

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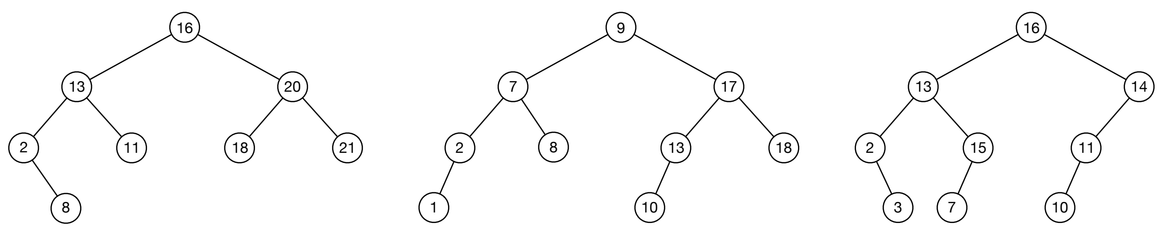
Reading

Introduction to Algorithms, Cormen, Rivest, Leisersons and Stein (CLRS): Chapter 12 excluding 12.4.

Exercises

1 Simulation and Properties

1.1 [w] Which of the following trees are binary search trees?



(a) (b) (c)

1.2 [w] Where are the elements with respectively the smallest and largest key located in a binary search tree?

1.3 [w] Consider the set of keys $\{1, 4, 5, 10, 16, 17, 21\}$. Draw binary search trees of height 2, 3, 4, 5, and 6 containing these keys.

1.4 [w] Specify the pre-order, in-order og post-order sequence of keys for the tree in (b).

1.5 Compare the heap property and the search tree properties.

1.6 Write pseudo code for an iterative version of inorder traversal.

1.7 Show that if a node v has 2 children, then its successor has no left child and its predecessor has no right child. Assume for simplicity that all keys are distinct. *Hint:* prove it by contradiction.

2 Leafs and Heights Let T be a binary tree with n vertices and root v.

2.1 Give a recursive algorithm that given v computes the number of leafs in T. Write pseudo code for your solution.

2.2 Give a recursive algorithm that given v computes the height of T. Write pseudo code for your solution.

2.3 [†] Implement your solution to compute the height.

3 More Recursion on Trees (Exam 2011) This exercise is about rooted binary trees. Each node x has fields $x.parent$, $x.left$, and $x.right$ denoting the parent, left child, and right child of x. For the root $root$, $root.parent = null$. Furthermore, we also store a field $x.size$ containing an integer. Consider the following algorithm.

```
ZERO(x)
if x ≠ null then
    ZERO(x.left)
    ZERO(x.right)
end if
```

3.1 Analyze the running time of the procedure ZERO(x) as a function of n, where x is the root node in a tree with n nodes.

3.2 Let $T(x)$ denote the subtree of the tree rooted at x and let $|T(x)|$ denote the number of nodes in $T(x)$. Give an algorithm, INITSIZE(x), that given the root node x of a tree sets $y.size$ to be $|T(y)|$ for all nodes y in the tree. Write your algorithm in pseudocode and analyse the running time of your algorithm as a function of n, where n is the number of nodes in the tree.

3.3 Given a node x with a child y of x, we say that the edge (x, y) is *red* if $|T(x)|$ is at least twice as large as $|T(y)|$. Give a recursive algorithm, REDEDGE(x), that given the root node, computes the number of red edges in the tree. Write your algorithm in pseudocode and analyse the running time of your algorithm as a function of n, where n is the number of nodes in the tree.

3.4 Analyse and give an upper bound in O-notation on the maximum number of red edges on any path from the root of the tree to a leaf.

4 Traversal of Binary Search Trees

4.1 Give an algorithm that given a binary search tree T with a key in each vertex, determines if T satisfies the binary search tree property.

4.2 Give an algorithm that given a binary search tree T constructs a *reversed binary search tree* T^R. T^R should be a binary search tree with the same keys as T. For each vertex v in T^R the vertices in the left subtree must be $\geq v$ and the keys in the right subtree must be $\leq v$.

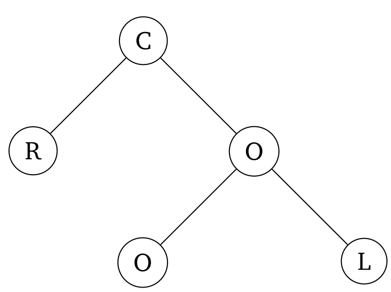
4.3 [∗] Give an algorithm that given two binary search trees T_1 and T_2 constructs a single binary search tree with all the elements from both T_1 and T_2.

5 Perfectly Balanced Binary Search Trees Let A be a sorted array of $n = 2^{h+1} - 1$ distinct numbers. Give a sequence of insertions of the numbers in A into a binary search tree T such that T becomes a complete binary search tree of height h.

6 Pre-Order Traversal [†] Implement a recursive algorithm for pre-order traversal of a binary tree.

7 Even More Recursion on Trees (Exam 2010) This exercise is about rooted binary trees. Each node x has fields $x.parent$, $x.left$, and $x.right$ denoting the parent, left child, and right child of x. For the root *root*, $root.parent = null$. Furthermore, we also store a field $x.label$ containing a single character. Consider the following algorithm and binary tree.

```
PrintTree(x)
if x ≠ null then
   print x.color
   if x.left ≠ null then
      PrintTree(x.left)
   end if
   if x.right ≠ null then
      PrintTree(x.right)
   end if
end if
```



7.1 If x is the root of the above tree, then PRINTTREE(x) outputs "CROOL". Explain how to modify PRINTTREE to print "COLOR" instead.

7.2 Give a recursive algorithm, INTERNAL(x), that given the root node x of a tree computes the number of internal nodes in the tree. Write your algorithm in pseudocode and analyse the running time of your algorithm as a function of n, where n is the number of nodes in the tree.

7.3 We say that a tree has an R-path if there is a root-to-leaf path consisting of only nodes labeled R. Give a recursive algorithm, R-PATH(x), that given the root node x of a tree determines if the tree has an R-path. Write your algorithm in pseudocode and analyse the running time of your algorithm as a function of n, where n is the number of nodes in the tree.