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Exercise 1.1

We construct a weighted, directed graph G, where the positions are the vertices. The ski slopes are directed edges from the start to the end position and weighted by their completion time.

Exercise 1.2

Algorithm We construct the graph G for the ski resort and run Dijkstra's shortest path algorithm from p_0. If the shortest path to p_{x-1} has length at most t, we output "yes, there is a trip" and otherwise "no, there is no trip."

Correctness A trip in a ski resort is a path in G, and the completion time of a trip is the length of the corresponding path. We compute the shortest path. Hence, this is at most t if and only if there is a trip with a completion time at most t.

Analysis We use $O(x + y)$ time to construct the graph and $O((x + y)\log x)$ time for Dijkstra's algorithm. In total, we use $O((x + y)\log x)$ time.

Exercise 1.3

We now also assign a level to each edge. Let G_ℓ denote the subgraph of G consisting of all edges with level at most ℓ. We binary search on the range of levels from $[1, x]$ to find the smallest ℓ such that the length of a shortest path from p_0 to p_{x-1} is at most t. Each step of the binary search proceeds as follows:

1. Construct graph G_ℓ.
2. Run Dijkstra on G_ℓ from p_0.
3. If the shortest path to p_{x-1} is at most t we lower ℓ and otherwise we increase ℓ.

We output the smallest ℓ determined by the binary search.

Correctness The algorithm computes the smallest ℓ such that G_ℓ has a shortest path from p_0 to p_{x-1} with total length at most t. By definition, this is the smallest level such that there is at trip from p_0 to p_{x-1} with completion time at most t.

Analysis Each binary search step uses $O((x + y)\log x)$ time. We binary search over a range of size x and thus use $O(\log x)$ binary search steps. In total, we use $O((x + y)\log^2 x)$ time.