02105 Algorithms and Data Structures 1 - MC Part May 2023

Der anvendes en scoringsalgoritme, som er baseret på "One best answer"

Dette betyder følgende:
Der er altid netop ét svar som er mere rigtigt end de andre
Studerende kan kun vælge ét svar per spørgsmål

The following approach to scoring responses is implemented and is based on "One best answer"
There is always only one correct answer – a response that is more correct than the rest
Students are only able to select one answer per question

For each question below, mark whether or not the statement is correct.

Select the correct answers

	Yes	No
$5n^3 + \frac{n^4}{100} = \Theta(n^3)$		X
$2^{\frac{1}{2}\log n} = O(\sqrt{n})$	X	
$4 \cdot \left(2\sqrt{n} + \log(n^4)\right) = \Omega(n^{1/3})$	X	
$\sqrt{\log n} + n^{1/5} = O(\log n)$		X
$\left(4n + \frac{1}{5}n^2 + \log n\right) \cdot n = \Omega(n^3)$	X	

Mark the running time in O-notation of each of the following algorithms as a function of n. Mark the best bound.

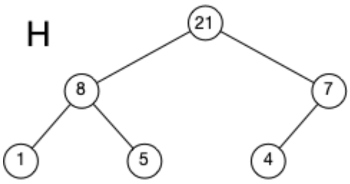
ALG1(n)
$t = 0$
for $i = 1$ **to** $2n\lceil \log n \rceil$ **do**
  for $j = 1$ **to** $\lceil n/2 \rceil$ **do**
    $t = t + 1$
  end for
end for

ALG2(n)
$t = 1$
$i = 1$
while $i \leq n^2$ **do**
  $i = 2 \cdot i$
end while

ALG3(n)
if $n \leq 1$ **then**
  return 1
else
  return ALG3$(n-2) + n$
end if

Select the correct answers	$O(1)$	$O(\log n)$	$O(\sqrt{n})$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(n^2 \log n)$	$O(n^3)$	$O(2^n)$
Alg1	○	○	○	○	○	○	☒	○	○
Alg2	○	☒	○	○	○	○	○	○	○
Alg3	○	○	○	☒	○	○	○	○	○

Consider the following binary heap H.

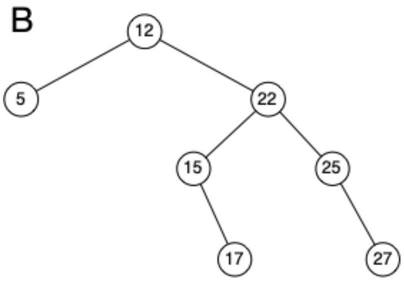


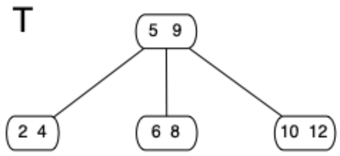
Let H_1 be the result of applying an Extract-Max operation on H and let H_2 be the result of applying an Insert operation with key 29 on H. Construct the arrays representations of H, H_1, and H_2 and state the content of each of the following entries (recall that a heap with n items in the array representation consists of an array of length n+1 where index 0 is not used).

Select the correct answers

	1	4	5	7	8	21	29
H[1]	○	○	○	○	○	X	○
H[2]	○	○	○	○	X	○	○
H[3]	○	○	○	X	○	○	○
H[5]	○	○	X	○	○	○	○
$H_1[1]$	○	○	○	○	X	○	○
$H_1[2]$	○	○	X	○	○	○	○
$H_1[3]$	○	○	○	X	○	○	○
$H_1[5]$	○	X	○	○	○	○	○
$H_2[1]$	○	○	○	○	○	○	X
$H_2[2]$	○	○	○	○	X	○	○
$H_2[3]$	○	○	○	○	○	X	○
$H_2[5]$	○	○	X	○	○	○	○

Consider the following binary search tree B and the 2-3 tree T.





Let B_1 be the result of applying an Insert operation with key 13 on B and let B_2 be the result of applying a Delete operation with key 22 on B. Furthermore, let T_1 be the result of applying an Insert operation with key 7 on T. Answer the following questions.

Select the correct answers	0	1	2	3	4	5	6	7	8
The height of B_1 is				X					
The height of B_2 is				X					
The height of T_1 is			X						
The number of leaves in B_1 is					X				
The number of leaves in B_2 is				X					
The number of nodes with 1 keys in T_1 is						X			
The number of nodes with 2 keys in T_1 is			X						

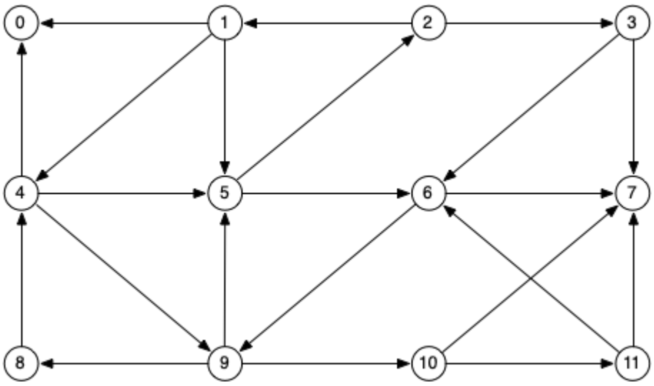
Consider the following data structures for representing a set of integers. We are interested in supporting the operation Min-Diff(), that returns a smallest difference between two integers in the set. For instance, if a data structure stores the set {32, 6, 18, 7, 2} then Min-Diff() should return 1 since the smallest difference between integers in the set is 7-6 = 1. For each data structure below mark the runtime of the Min-Diff() operation in O-notation as a function of n, where n is the number of elements in the data structure. Mark the best bound

Select the correct answers

	$O(1)$	$O(\log n)$	$O(\sqrt{n})$	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(2^n)$
Sorted array	○	○	○	X	○	○	○
Max-heap	○	○	○	○	X	○	○
Min-heap	○	○	○	○	X	○	○
Array (not sorted)	○	○	○	○	X	○	○
Binary search tree	○	○	○	X	○	○	○

Graph Search

Consider the following graph G.



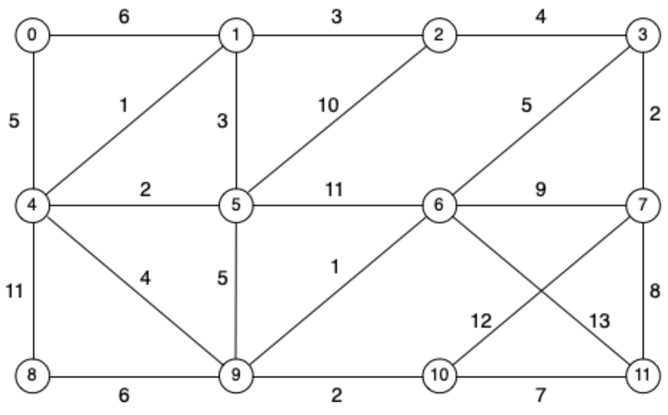
Construct the DFS tree T_1 for G when starting in vertex 5. Assume that each adjacency list is sorted in increasing order and T_1 is rooted in vertex 5. Answer the following questions.

Select the correct answers	1	2	3	4	5	6	7	8	9
The depth of T_1 is	○	○	○	○	○	○	☒	○	○
The number of leaves of T_1 is	○	○	○	○	☒	○	○	○	○
The maximum number of children of a node in T_1 is	○	☒	○	○	○	○	○	○	○

Construct the BFS tree T_2 for G when starting in vertex 5. Assume that each adjacency list is sorted in increasing order and T_2 is rooted in vertex 5. Answer the following questions.

Select the correct answers	1	2	3	4	5	6	7	8	9
The depth of T_2 is	○	○	○	☒	○	○	○	○	○
The number of leaves of T_2 is	○	○	○	○	○	☒	○	○	○
The maximum number of children of a node in T_2 is	○	☒	○	○	○	○	○	○	○

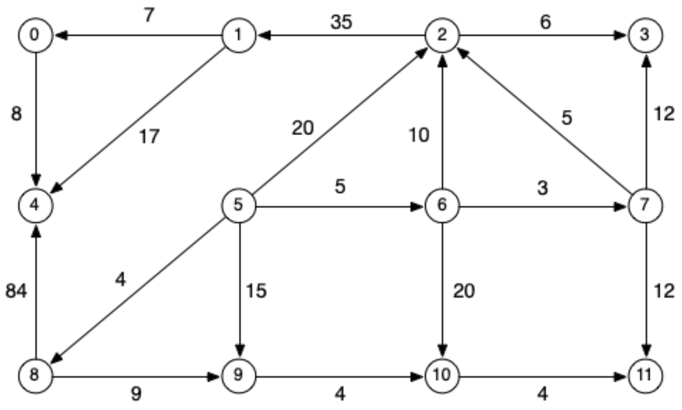
Consider the following graph.



Construct a minimum spanning tree T for the graph. Assume that T is rooted in vertex 5 and answer the following questions.

Select the correct answers	1	2	3	4	5	6	7	8	9
The depth of T is	○	○	○	○	○	○	○	○	○
The number of leaves of T is	○	○	○	○	○	○	○	○	○
The maximum number of children of a node in T is	○	○	○	○	○	○	○	○	○

Consider the following graph



Construct a shortest path tree T for the graph when starting at vertex 5. Assume that T is rooted in vertex 5 and answer the following questions

Select the correct answers	1	2	3	4	5	6	7	8	9
The depth of T is	○	○	○	○	○	○	○	○	○
The number of leaves in T is	○	○	○	○	○	○	○	○	○
The maximum number of children of a node in T is	○	○	○	○	○	○	○	○	○