

Reading Material

KT Chapter 4 (Divide and Conquer), section 4.1 (A First Recurrence: The Mergesort Algorithm), 4.2 (Further Recurrence Relations), and 4.5 (Integer Multiplication).

If you have another edition of the book than the international one from 2014 *Divide and Conquer* might be in a different chapter (e.g. 5), but the subsections are the same. In that case you should of course also solve the corresponding exercises (the numbers within the chapters are the same, e.g. if *Divide and Conquer* is chapter 5 in our book, you should solve exercise 5.1 instead of 4.1).

Exercises

1 Recurrences Use both the *recursion tree method* and the *(partial) substitution method* to solve each of the following recurrences.

$$1.1 \quad T(n) \leq \begin{cases} 2T(n/4) + c\sqrt{n} & \text{for } n > 4 \\ c & \text{otherwise} \end{cases}$$

$$1.2 \quad T(n) \leq \begin{cases} 2T(n/4) + cn & \text{for } n > 4 \\ c & \text{otherwise} \end{cases}$$

$$1.3 \quad T(n) \leq \begin{cases} 2T(n/4) + cn^2 & \text{for } n > 4 \\ c & \text{otherwise} \end{cases}$$

$$1.4 \quad T(n) \leq \begin{cases} T(\frac{3n}{4}) + cn & \text{for } n > 4 \\ c & \text{otherwise} \end{cases}$$

$$1.5 \quad T(n) \leq \begin{cases} T(n/2) + T(n/3) + T(n/6) + cn & \text{for } n > 6 \\ c & \text{otherwise} \end{cases}$$

2 Median Solve KT exercise 4.1.

3 Divide-and-conquer on trees Solve KT exercise 4.6.

4 Divide-and-conquer on grid graphs Solve KT exercise 4.7.

5 Kattis exercises To get familiar with the Kattis online judge system solve the following two exercises on Kattis (you first need to make a Kattis profile).

- Tower Construction: <https://open.kattis.com/problems/tornbygge>
- Falling Apart: <https://open.kattis.com/problems/fallingapart>

We will use Kattis for our programming exercises during the course.

Puzzle of the week: The Switch The hangman summons his 100 prisoners, announcing that they may meet to plan a strategy, but will then be put in isolated cells, with no communication. He explains that he has set up a switch room which contains a single switch, which is either on or off. It is not known to the prisoners whether the switch initially is on or off. Also, the switch is not connected to anything, but a prisoner entering the room may see whether the switch is on or off (because the switch is up or down). Every once in a while, the hangman will let one arbitrary prisoner into the switch room. The prisoner may throw the switch (on to off, or vice versa), or leave the switch unchanged. Nobody but the prisoners will ever enter the switch room. The hangman promises

to let any prisoner enter the room from time to time, arbitrarily often. That is, eventually, each prisoner has been in the room at least once, twice, a thousand times, any number you want. At any time, any prisoner may declare "We have all visited the switch room at least once". If the claim is correct, all prisoners will be released. If the claim is wrong, the hangman will execute his job (on all the prisoners). What's the strategy?