String Matching

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CLRS 32

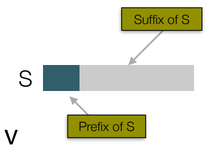
String Matching

- String matching problem:
  - string T (text) and string P (pattern) over an alphabet Σ.
  - |T| = n, |P| = m.
  - Report all starting positions of occurrences of P in T.

P = a b a b a c a

T = b a c b a b a b a b a b a c a b

Strings



- ε: empty string
- prefix/suffix: v=xy:
  - x *prefix* of v, if y ≠ ε x is a *proper prefix* of v
  - y *suffix* of v, if y ≠ ε x is a *proper suffix* of v.
- Example: S = aabca
  - The suffixes of S are: aabca, abca, bca, ca and a.
  - The strings abca, bca, ca and a are proper suffixes of S.

String Matching

- Knuth-Morris-Pratt (KMP)
- Finite automaton

A naive string matching algorithm

| b | a | c | b | a | b | a | b | a | b | a | b | a | c | a | b |

a b a b a c a
a b a b a c a
a b a b a c a
a b a b a c a
a b a b a c a
a b a b a c a
a b a b a c a
a b a b a c a
a b a b a c a
a b a b a c a

Improving the naive algorithm

P = a a a b a b a

T = a a a b a a [redacted]

a a a b a b a
a a a b a b a

Exploiting what we know from pattern

P = a b a b a c a

T = a b a b a a x [redacted]
a b a b a c a — How much should we shift the pattern? 5
a b a b a c a a b a c a

T = a b a b a b x [redacted]
a b a b a c a — How much should we shift the pattern? 2
a b a b a c a

T = a b a b a c x [redacted]
a b a b a c a — How much should we shift the pattern? 0
a b a b a c a

Exploiting what we know from pattern

P = a b a b a c a

T = a b a b a a x [redacted]
a b a b a c a — Which character in the pattern should we compare to x? 2
a b a b a c a

T = a b a b a b x [redacted]
a b a b a c a — Which character in the pattern should we compare to x? 5
a b a b a c a

T = a b a b a c x [redacted]
a b a b a c a — Which character in the pattern should we compare to x? 7
a b a b a c a

Exploiting what we know from pattern

P = a b a b a c a

T = a b a b a a x

a b a b a c a — How much can we “reuse”? — 1

a b a b a c a

T = a b a b a b x

a b a b a c a — How much can we “reuse”? — 4

a b a b a c a

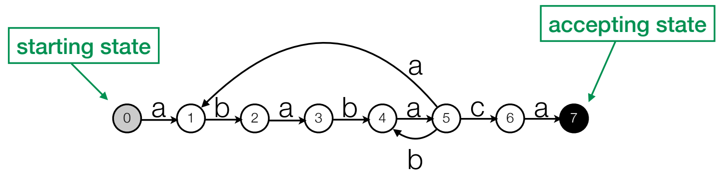
T = a b a b a c x

a b a b a c a — How much can we “reuse”? — 6

a b a b a c a

Finite Automaton

- Finite automaton: alphabet Σ = {a,b,c}. P = ababaca.



Finite Automaton

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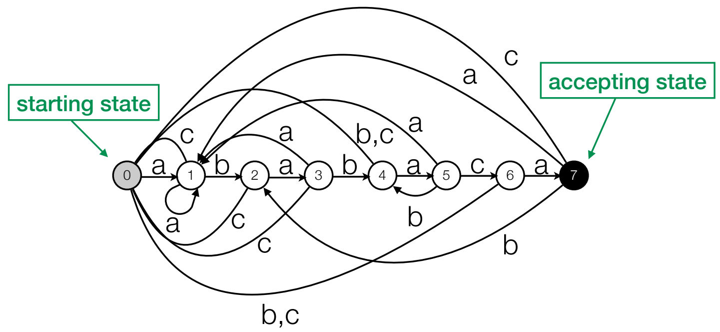

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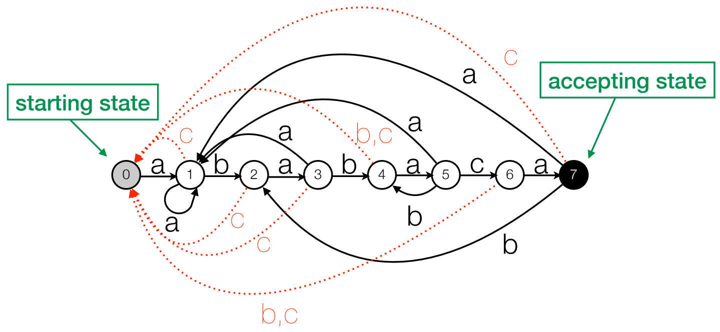

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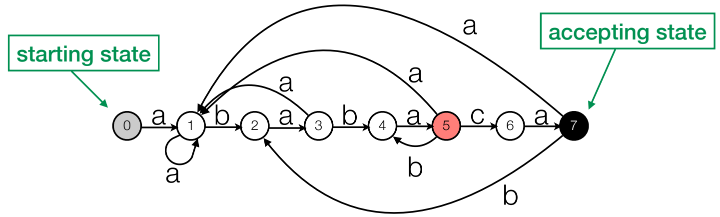

- State j: arc with character α goes to state i ≤ j +1 such that $P[1..i]$ is the longest prefix of P that is a suffix of $P[1\ldots j] \cdot \alpha$.

Finite Automaton

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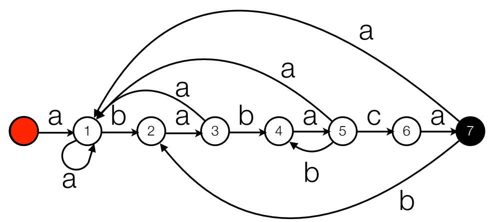
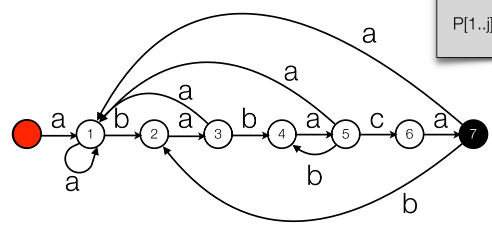


T = b a c b a b a b a b a b a c a b

Finite Automaton

- Finite automaton: alphabet Σ = {a,b,c}. P = ababaca.

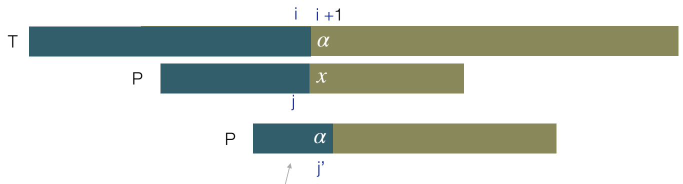
If we are in state j after reading T[1..i], then P[1..j] is the longest prefix of P that is a suffix of T[1..i].



T = b a c b a b a b a b a b a c a b

Finite Automaton

If we are in state j after reading T[1..i], then
P[1..j] is the longest prefix of P that is a suffix of T[1..i].



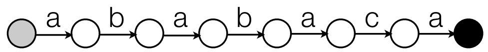
P[1..j'] longest prefix of P that is a suffix of T[1..i+1].

P[1..j'-1] is a prefix of P[1..j].

$\Rightarrow$ P[1..j'] longest prefix of P that is a suffix of P[1..j] $\cdot\ \alpha$.

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.



Finite Automaton Construction

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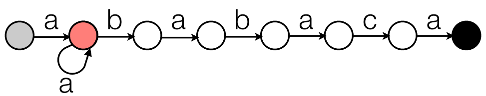


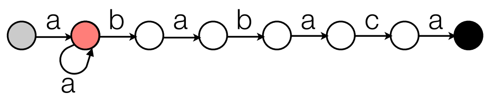
read ‘a’? **longest prefix of P that is a proper suffix of ‘aa’ = ‘a’**

Matched until now: a a

P: a b a b a c a

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.



read ‘c’? **longest prefix of P that is a proper suffix of ‘ac’ = ‘ ’**

Matched until now: a c

P: a b a b a c a

Finite Automaton Construction

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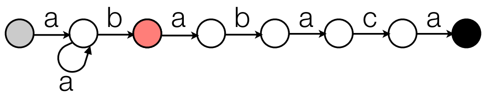

read ‘b’? **longest prefix of P that is a proper suffix of ‘abb’ = ‘ ’**

Matched until now: a b b

P: a b a b a c a

Finite Automaton Construction

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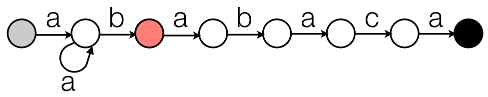

read ‘c’? **longest prefix of P that is a proper suffix of ‘abc’ = ‘ ’**

Matched until now: a b c

P: a b a b a c a

Finite Automaton Construction

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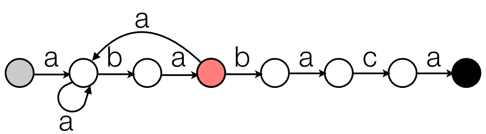


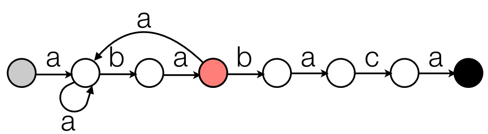
read ‘a’? **longest prefix of P that is a proper suffix of ‘abaa’ = ‘a’**

Matched until now: a b a a

P: a b a b a c a

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.



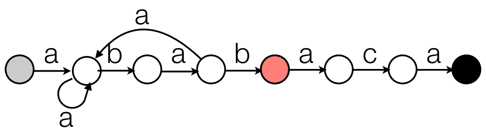
read ‘c’? **longest prefix of P that is a proper suffix of ‘abac’ = ‘ ’**

Matched until now: a b a c

P: a b a b a c a

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.



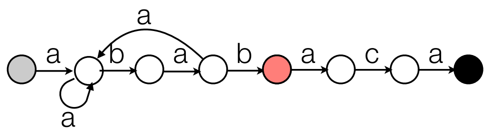
read ‘b’? **longest prefix of P that is a proper suffix of ‘ababb’ = ‘ ’**

Matched until now: a b a b b

P: a b a b a c a

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.



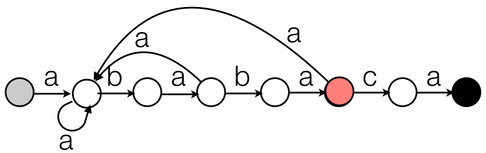
read ‘c’? **longest prefix of P that is a proper suffix of ‘ababc’ = ‘ ’**

Matched until now: a b a b c

P: a b a b a c a

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.



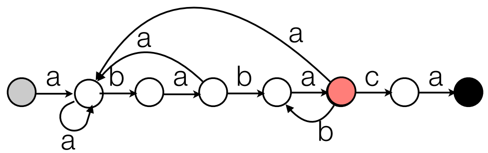
read ‘a’? **longest prefix of P that is a proper suffix of ‘ababaa’ = ‘a’**

Matched until now: a b a b a a

P: a b a b a c a

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.



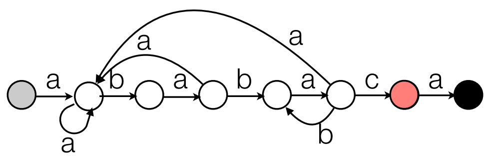
read ‘b’? **longest prefix of P that is a proper suffix of ‘ababaa’ = ‘abab’**

Matched until now: a b a b a b

P: a b a b a c a

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.

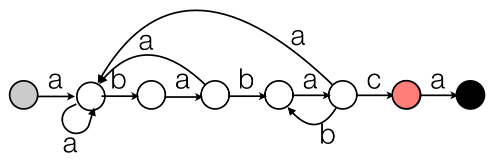


read ‘b’?

longest prefix of P that is a proper suffix of ‘ababacb’ = ‘ ’

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.

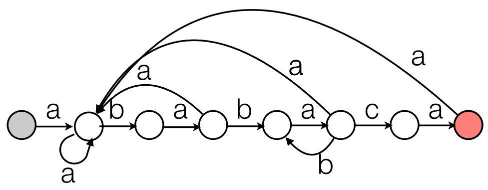


read ‘c’?

longest prefix of P that is a proper suffix of ‘ababacc’ = ‘ ’

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.

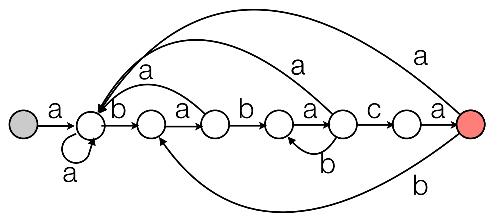


read ‘a’?

longest prefix of P that is a proper suffix of ‘ababacaa’ = ‘a’

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.

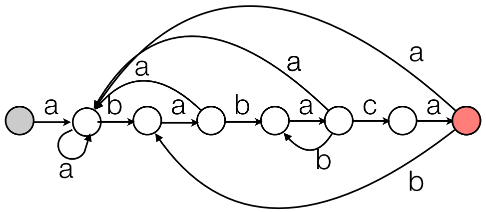


read ‘b’?

longest prefix of P that is a proper suffix of ‘ababacab’ = ‘ab’

Finite Automaton Construction

- Finite automaton: alphabet Σ = {a,b,c}. P= ababaca.

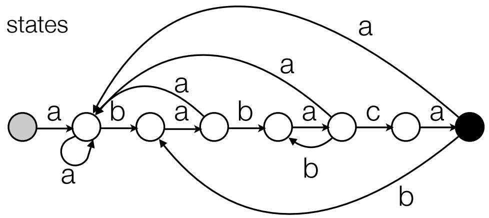


read ‘c’?

longest prefix of P that is a proper suffix of ‘ababacac’ = ‘ ’

Finite Automaton

- Finite automaton:
  - Q: finite set of states
  - $q_0 \in Q$: start state
  - $A \subseteq Q$: set of accepting states
  - Σ: finite input alphabet
  - δ: transition function

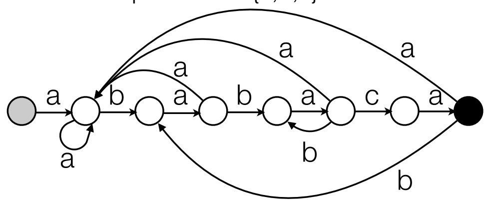


- Matching time: O(n)
- Preprocessing time: $O(m^3|\Sigma|)$. (Can be done in $O(m|\Sigma|)$).
- Total time: $O(n + m|\Sigma|)$

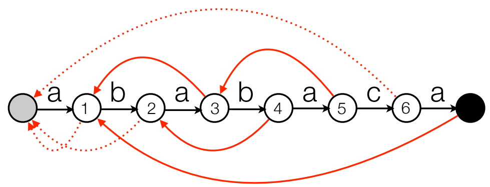
KMP

KMP

- Finite automaton: alphabet Σ = {a,b,c}. P = ababaca.

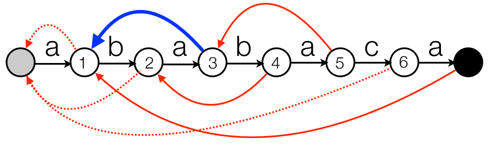


- KMP: Can be seen as finite automaton with *failure links*:



KMP

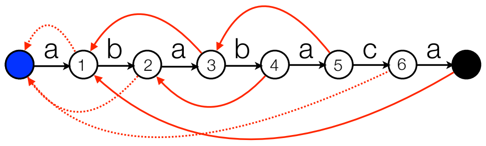
- KMP: Can be seen as finite automaton with *failure links*:
  - longest prefix of P that is a suffix of what we have *matched* until now (ignore the mismatched character).



longest prefix of P that is a proper suffix of 'aba'

KMP matching

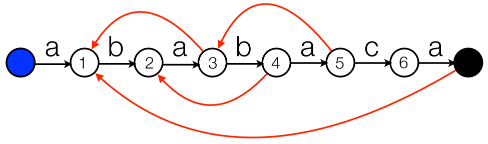
- KMP: Can be seen as finite automaton with *failure links*:
  - longest prefix of P that is a suffix of what we have *matched* until now.



T = b a c b a b a b a b a b a c a b

KMP

- KMP: Can be seen as finite automaton with *failure links*:
  - longest prefix of P that is a proper suffix of what we have *matched* until now.
  - can follow several failure links when matching one character:



T = a b a b a a

KMP Analysis

- Analysis. |T| = n, |P| = m.
  - How many times can we follow a forward edge?
  - How many backward edges can we follow (compare to forward edges)?
  - Total number of edges we follow?
  - What else do we use time for?

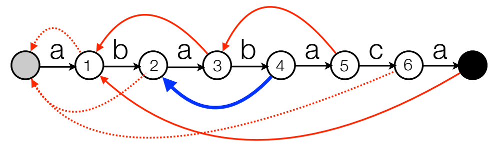
KMP Analysis

- Lemma. The running time of KMP matching is O(n).
  - Each time we follow a forward edge we read a new character of T.
  - #backward edges followed ≤ #forward edges followed ≤ n.
  - If in the start state and the character read in T does not match the forward edge, we stay there.
  - Total time = #non-matched characters in start state + #forward edges followed + #backward edges followed ≤ 2n.

Computation of failure links

If we are in state j after reading T[1..i], then P[1..j] is the longest prefix of P that is a suffix of T[1..i].

- Failure link: longest prefix of P that is a proper suffix of what we have *matched* until now.



longest proper prefix of P that is a suffix of ‘abab'

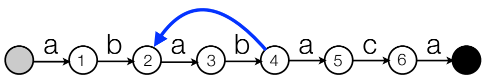
Matched until now: a b a b

a b a b a c a

Computation of failure links

- Failure link: longest prefix of P that is a proper suffix of what we have *matched* until now.
- Computing failure links: Use KMP matching algorithm.

longest prefix of P that is a proper suffix of ‘abab'

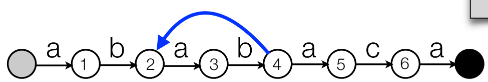


Computation of failure links

- Failure link: longest prefix of P that is a proper suffix of what we have *matched* until now.
- Computing failure links: Use KMP matching algorithm.

If we are in state j after reading T[1..i], then P[1..j] is the longest prefix of P that is a suffix of T[1..i].

longest prefix of P that is a suffix of ‘bab'

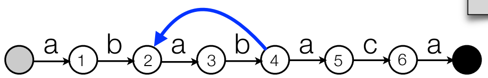


Computation of failure links

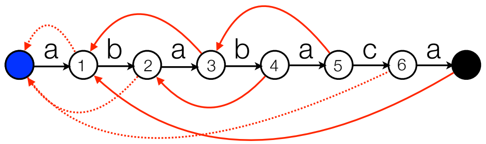
- Failure link: longest prefix of P that is a proper suffix of what we have *matched* until now.
- Computing failure links: Use KMP matching algorithm.

If we are in state j after reading T[1..i], then P[1..j] is the longest prefix of P that is a suffix of T[1..i].

longest prefix of P that is a suffix of 'bab'

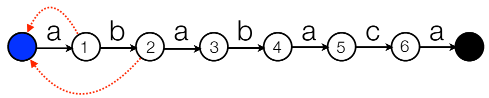


Can be found by using KMP to match 'bab'



Computation of failure links

- Computing failure links: As KMP matching algorithm (only need failure links that are already computed).
- Failure link: longest prefix of P that is a proper suffix of what we have *matched* until now.



T = b

Computation of failure links

- Computing failure links: As KMP matching algorithm (only need failure links that are already computed).
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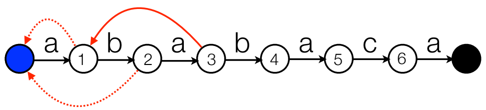

T = b a

Computation of failure links

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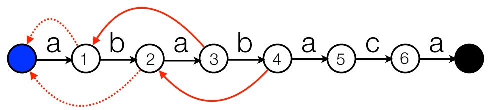

T = b a b

Computation of failure links

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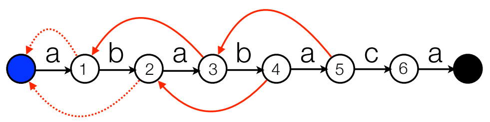

T = b a b a

Computation of failure links

- Computing failure links: As KMP matching algorithm (only need failure links that are already computed).
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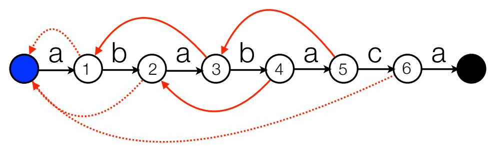
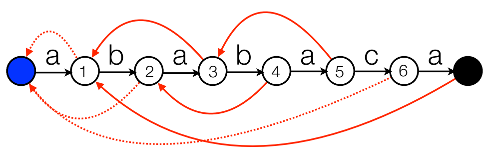


T = b a b a c

Computation of failure links

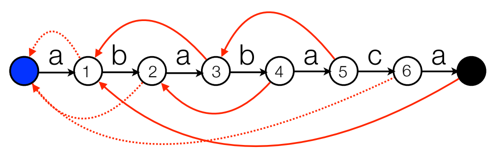
- Computing failure links: As KMP matching algorithm (only need failure links that are already computed).
- Failure link: longest prefix of P that is a proper suffix of what we have *matched* until now.



T = b a b a c a

Computation of failure links

- Computing failure links: As KMP matching algorithm (only need failure links that are already computed).
- Failure link: longest prefix of P that is a proper suffix of what we have *matched* until now.



1 2 3 4 5 6 7

P = ■ b a b a c a

KMP

- Computing π: As KMP matching algorithm (only need π values that are already computed).
- Running time: O(n + m):
  - Lemma. Total number of comparisons of characters in KMP is at most 2n.
  - Corollary. Total number of comparisons of characters in the preprocessing of KMP is at most 2m.

KMP: the π array

- π array: A representation of the failure links.
- Takes up less space than pointers.

i	1	2	3	4	5	6	7
π[i]	0	0	1	2	3	0	1