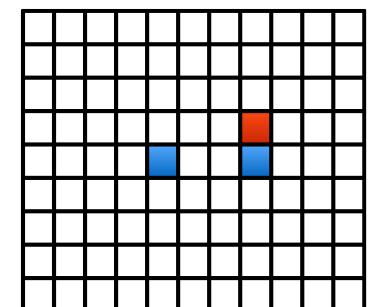


Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

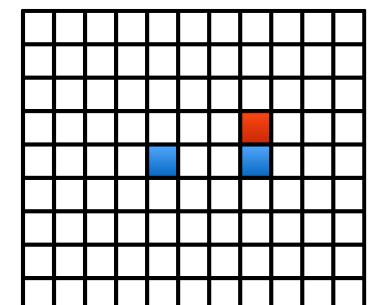


Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

```
Subset-Sum(n, W)
    Array M[0...n][0...W]
    Initialize M[0][w] = 0 for each w = 0, 1, ..., W
    for i = 1 to n
        for w = 0 to W
            if w < wi
                M[i][w] = M[i-1][w]
            else
                M[i][w] = max(M[i-1][w], wi + M[i-1][w-wi])
    return M[n, W]
```



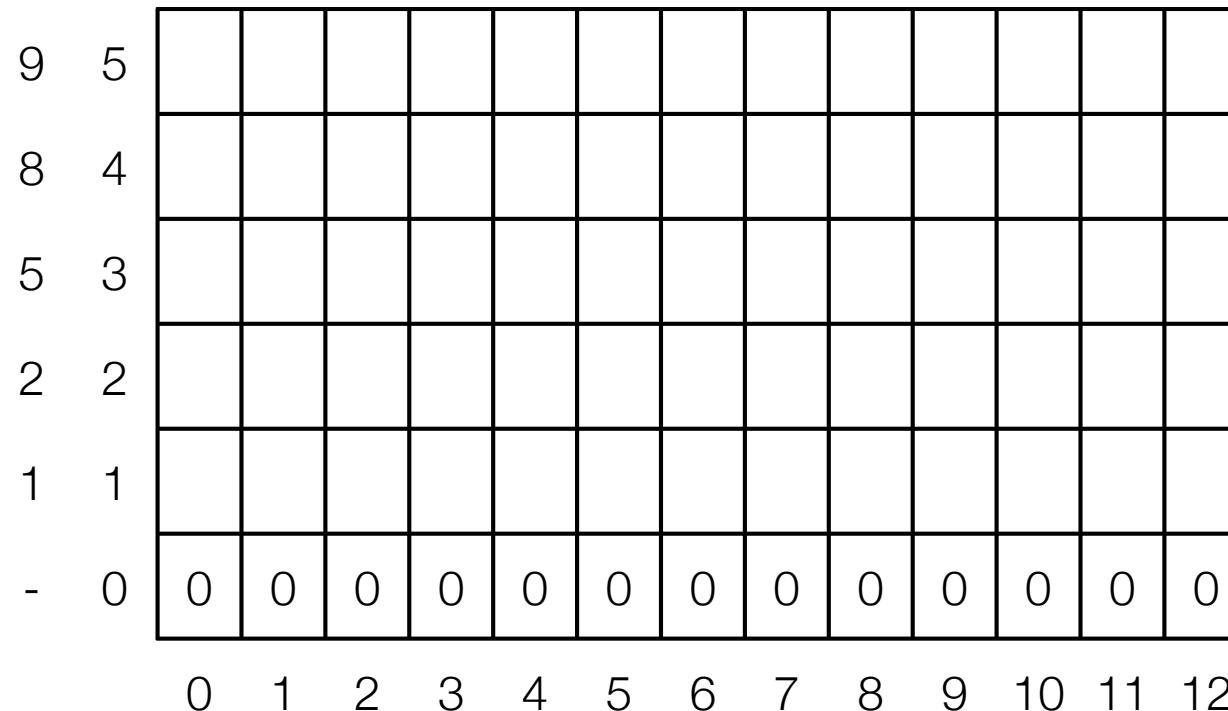
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$



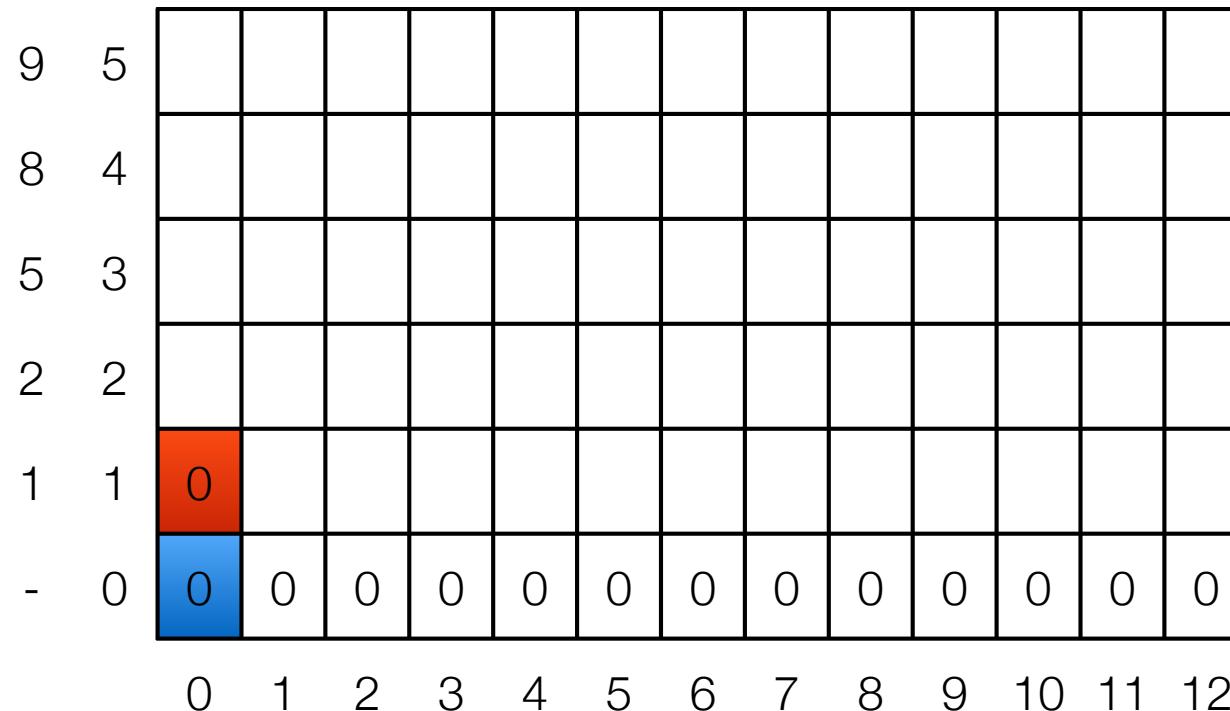
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

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- $\{1, 2, 5, 8, 9\}$ and $W = 12$



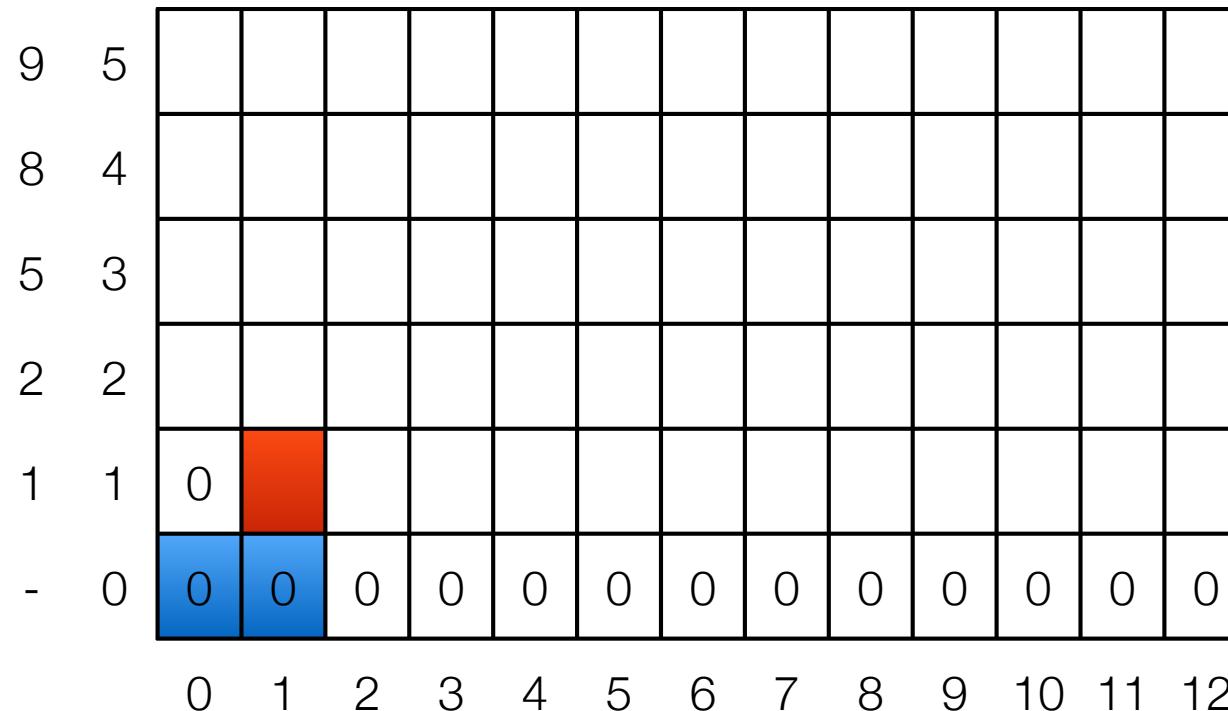
Subset Sum

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$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$



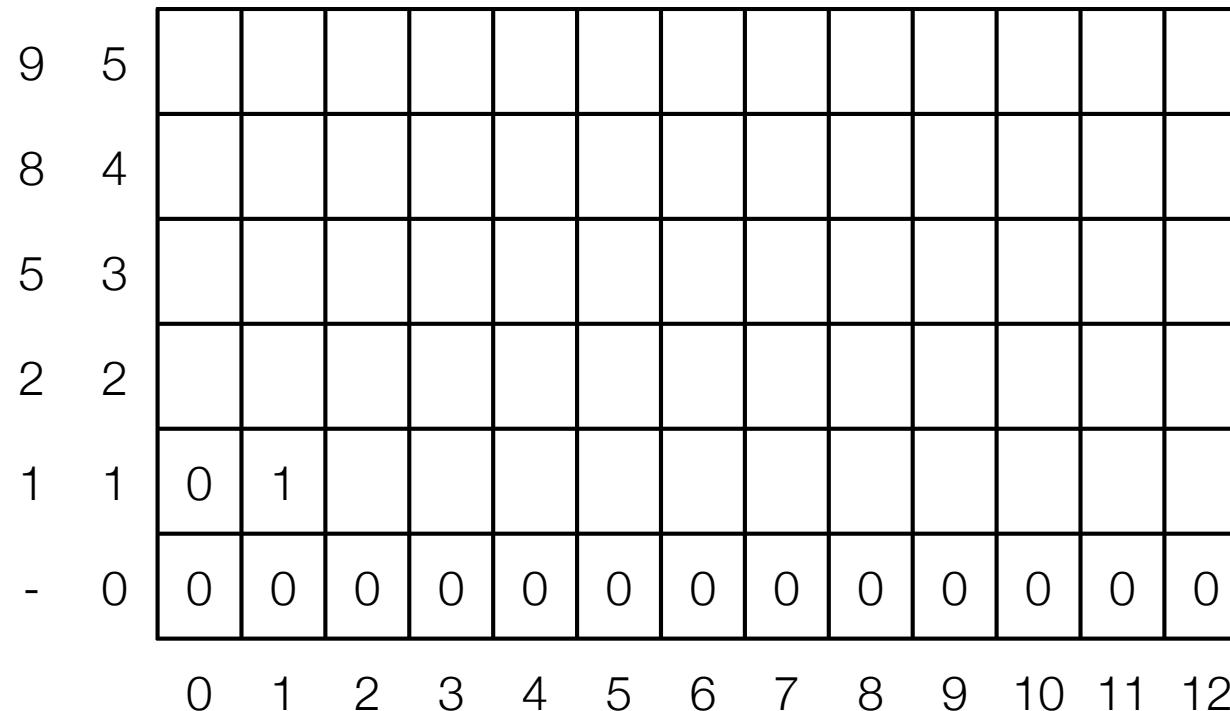
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$



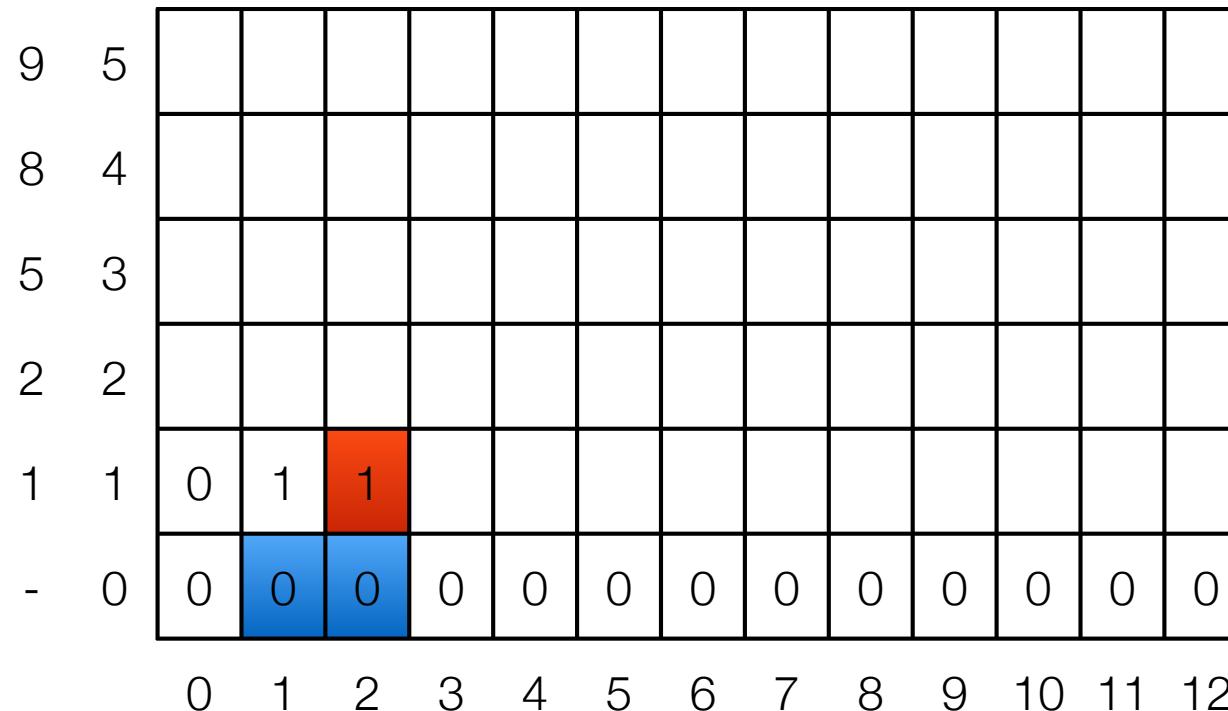
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

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Subset Sum

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- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$

9	5												
8	4												
5	3												
2	2												
1	1	0	1	1	1	1	1	1	1	1	1	1	
-	0	0	0	0	0	0	0	0	0	0	0	0	
	0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$

9	5												
8	4												
5	3												
2	2	0											
1	1	0	1	1	1	1	1	1	1	1	1	1	
-	0	0	0	0	0	0	0	0	0	0	0	0	
	0	1	2	3	4	5	6	7	8	9	10	11	12

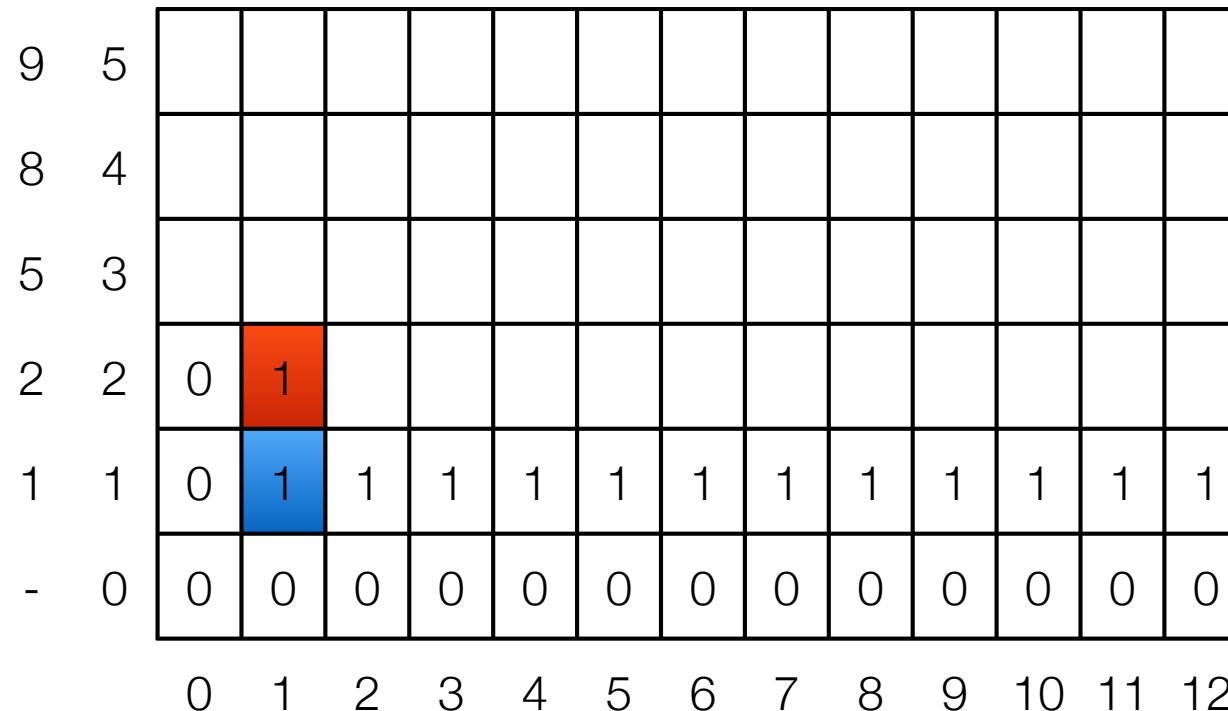
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$



Subset Sum

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$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$

9	5												
8	4												
5	3												
2	2	0	1	2									
2	2	0	1	1									
1	1	0	1	1	1	1	1	1	1	1	1	1	
-	0	0	0	0	0	0	0	0	0	0	0	0	
	0	1	2	3	4	5	6	7	8	9	10	11	12

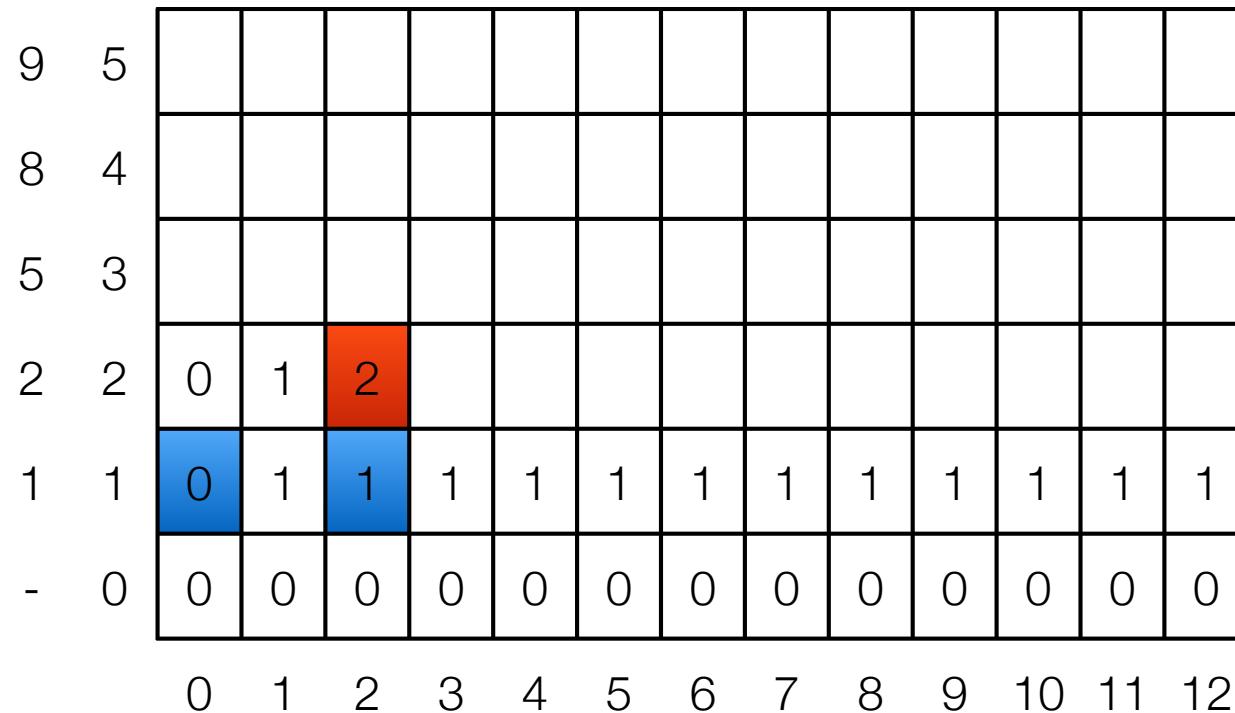
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$



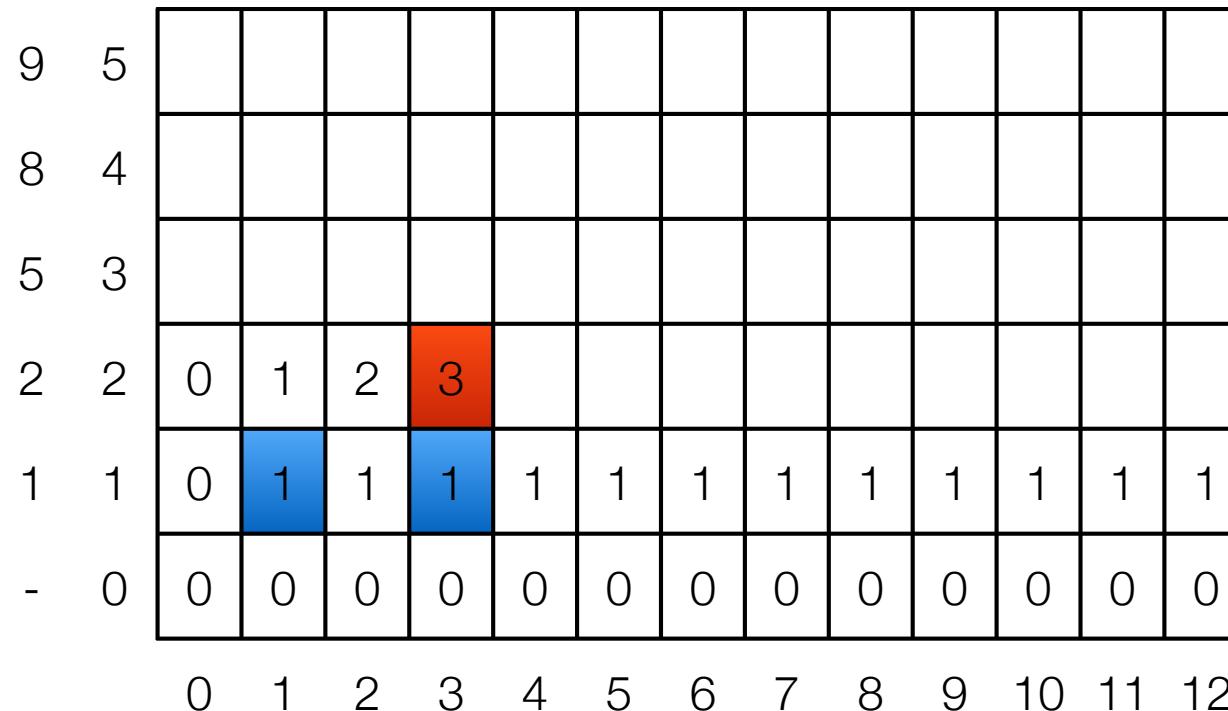
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$



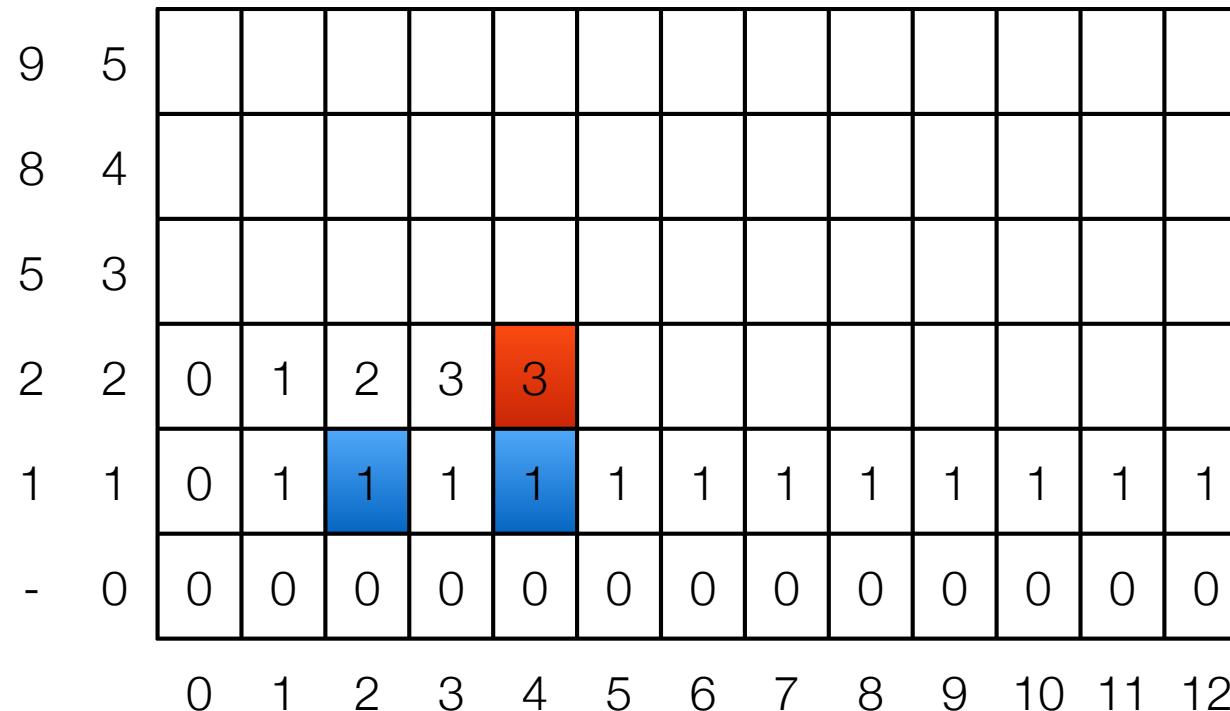
Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$



Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$

9	5											
8	4											
5	3											
2	2	0	1	2	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$

9	5											
8	4											
5	3	0	1	2	3	3	5					
2	2	0	1	2	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0

The table illustrates the dynamic programming solution for the Subset Sum problem. The columns represent the weight w from 0 to 12, and the rows represent the index i of the items from 0 to 5. The value at each cell (i, w) is $\text{OPT}(i, w)$. The values are color-coded: blue for $w < w_i$ (rows 0-2), orange for $w = w_i$ (row 3), and white for $w > w_i$ (rows 4-5). The final value $\text{OPT}(5, 12) = 3$ is highlighted in orange.

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$

9	5												
8	4												
5	3	0	1	2	3	3	5	6					
2	2	0	1	2	3	3	3	3	3	3	3	3	
1	1	0	1	1	1	1	1	1	1	1	1	1	
-	0	0	0	0	0	0	0	0	0	0	0	0	
	0	1	2	3	4	5	6	7	8	9	10	11	12

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$

9	5											
8	4											
5	3	0	1	2	3	3	5	6	7	8	8	8
2	2	0	1	2	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	2	3	4	5	6	7	8	9	10	11

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- $\{1, 2, 5, 8, 9\}$ and $W = 12$

9	5											
8	4	0	1	2	3	3	5	6	7	8	9	
5	3	0	1	2	3	3	5	6	7	8	8	8
2	2	0	1	2	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10

Subset Sum

- Recurrence:

$$\text{OPT}(i, w) = \begin{cases} \text{OPT}(i - 1, w) & \text{if } w < w_i \\ \max(\text{OPT}(i - 1, w), w_i + \text{OPT}(i - 1, w - w_i)) & \text{otherwise} \end{cases}$$

- Example

- {1, 2, 5, 8, 9} and W = 12

9	5	0	1	2	3	3	5	6	7	8	9	10	11	12
8	4	0	1	2	3	3	5	6	7	8	9	10	11	11
5	3	0	1	2	3	3	5	6	7	8	8	8	8	8
2	2	0	1	2	3	3	3	3	3	3	3	3	3	3
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
-	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0	1	2	3	4	5	6	7	8	9	10	11	12