

Reading Material

KT section 5.1, 5.2, and 5.5.

Exercises

1 Integer multiplication Show the first recursive calls to Karatsuba's algorithm (Recursive-Multiply) when the algorithm is run on input $x = 10110111$ and $y = 11001001$. That is, write what x_0 , x_1 , y_0 , and y_1 are, and write which binary numbers Recursive-Multiply is called with.

2 Recurrences Use both the recursion tree method and substitution method to solve each of the following recurrences. In all three cases the base case is $T(n) = c$ for $n \leq 2$.

2.1 $T(n) = T(\frac{3n}{4}) + cn.$

2.2 $T(n) = 2T(n/4) + c\sqrt{n}.$

2.3 $T(n) = 2T(n/4) + cn.$

3 Median Solve KT exercise 5.1.

4 Fraud detection Solve KT exercise 5.3.

5 Divide-and-conquer on trees Solve KT exercise 5.6.

6 Divide-and-conquer on grid graphs Solve KT exercise 5.7.

Puzzle of the week: The Switch The hangman summons his 100 prisoners, announcing that they may meet to plan a strategy, but will then be put in isolated cells, with no communication. He explains that he has set up a switch room which contains a single switch, which is either on or off. It is not known to the prisoners whether the switch initially is on or off. Also, the switch is not connected to anything, but a prisoner entering the room may see whether the switch is on or off (because the switch is up or down). Every once in a while, the hangman will let one arbitrary prisoner into the switch room. The prisoner may throw the switch (on to off, or vice versa), or leave the switch unchanged. Nobody but the prisoners will ever enter the switch room. The hangman promises to let any prisoner enter the room from time to time, arbitrarily often. That is, eventually, each prisoner has been in the room at least once, twice, a thousand times, any number you want. At any time, any prisoner may declare "We have all visited the switch room at least once". If the claim is correct, all prisoners will be released. If the claim is wrong, the hangman will execute his job (on all the prisoners). What's the strategy?