Reading material

We will introduce the paradigm *dynamic programming*. You should read KT Section 6.1, 6.2, and 6.3.

Exercises

1 [w] **Weighted interval scheduling** Solve the following weighted interval scheduling problem using both the recursive method with memoization and the iterative method. The intervals are given as triples (s_i, f_i, v_i): $S = \{(1,7,4),(10,12,2),(2,5,3),(8,11,4),(12,13,3),(3,9,5),(3,4,3),(4,6,3),(5,8,2),(4,13,6)\}$.

2 Predecessor using binary search In the weighted interval scheduling algorithm we used binary search to find compute $p(j)$ in the lecture. Explain how you can use binary search to find the predecessor of an element x in a sorted array (the predecessor of x is the largest element smaller than or equal to x).

3 Segmented least squares Write pseudo code for the following algorithms:

- An algorithm that solves the segmented least squares problem using recursion and memoization.
- An algorithm that computes segments of the solution (not just the value)

4 Independent set on a path Solve KT 6.1.

5 Job planning Solve KT 6.2.

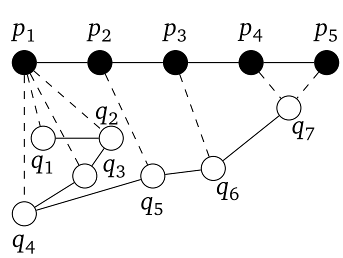
6 Office switching Solve KT 6.4.

7 Discrete Fréchet distance Consider Professor Bille going for a walk with his dog. The professor follows a path of points $p_1, \dots, p_n$ and the dog follows a path of points $q_1, \dots, q_m$. We assume that the walk is partitioned into a number of small steps, where the professor and the dog in each step either both move from p_i tp p_{i+1} and from q_j to q_{j+1}, respectively, or only one of them moves and the other one stays.

The goal is to find the smallest possible length L of the leash, such that the professor and the dog can move from p_1 and q_1, resp., to p_n and q_n. They cannot move backwards, and we only consider the distance between points. The distance L is also known as the discrete Fréchet distance.

We let $L(i, j)$ denote the smallest possible length of the leash, such that the professor and the dog can move from p_1 and q_1 to p_i and q_j, resp. For two points p and q, let $d(p, q)$ denote the distance between the them.

In the example below the dotted lines denote where Professor Bille (black nodes) and the dog (white nodes) are at time 1 to 8. The minimum leash length is $L = d(p_1, q_4)$.



7.1 Give a recursive formula for $L(i, j)$.

7.2 Give pseudo code for an algorithm that computes the length of the shortest possible leash. Analyze space and time usage of your solution.

7.3 Extend your algorithm to print out paths for the professor and the dog. The algorithm must return where the professor and the dog is at each time step. Analyze the time and space usage of your solution.