

Mandatory Exercise: Hashing

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1 Bipartite 2-Paths Let $G = (V_1 \cup V_2, E)$ be a bipartite graph, with $|V_1| + |V_2| = n$ nodes and $m \geq n$ edges. We want a data structure for G that supports the following operation on any pair of nodes v and u from V_1 .

- $\text{2-path}(v, u)$: Return yes if there is a path of length 2 between v and u and no otherwise.

Solve the following exercises.

- 1.1** Give a data structure that uses $O(|V_1|^2)$ space and supports fast 2-path queries. The query time should be $o(m)$.
- 1.2** Give a data structure that uses $O(m)$ space and supports fast 2-path queries. *Hint*: A good solution has a query time that depends on the degrees of the input nodes.

You do not need to consider preprocessing time.