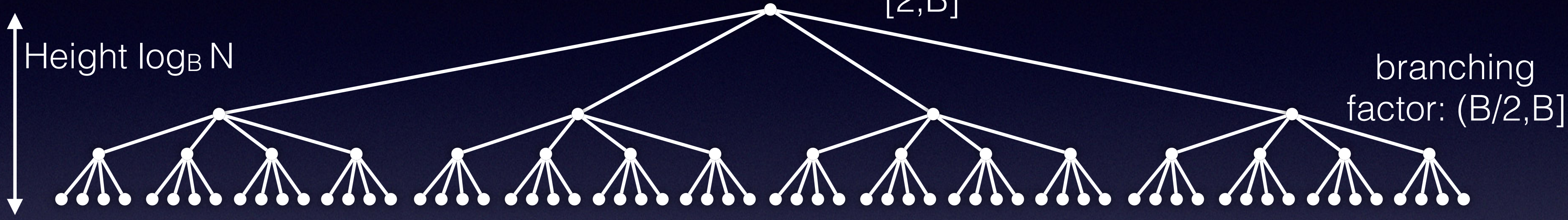
I/O data structures

B-trees and B^{ε}-trees

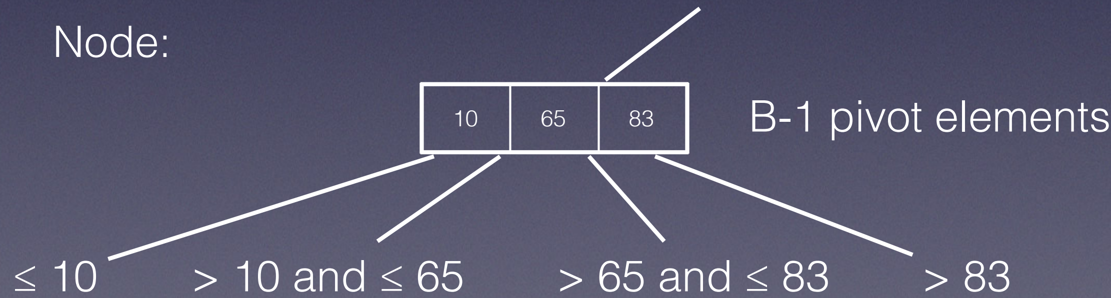
Inge Li Gørtz

B-trees

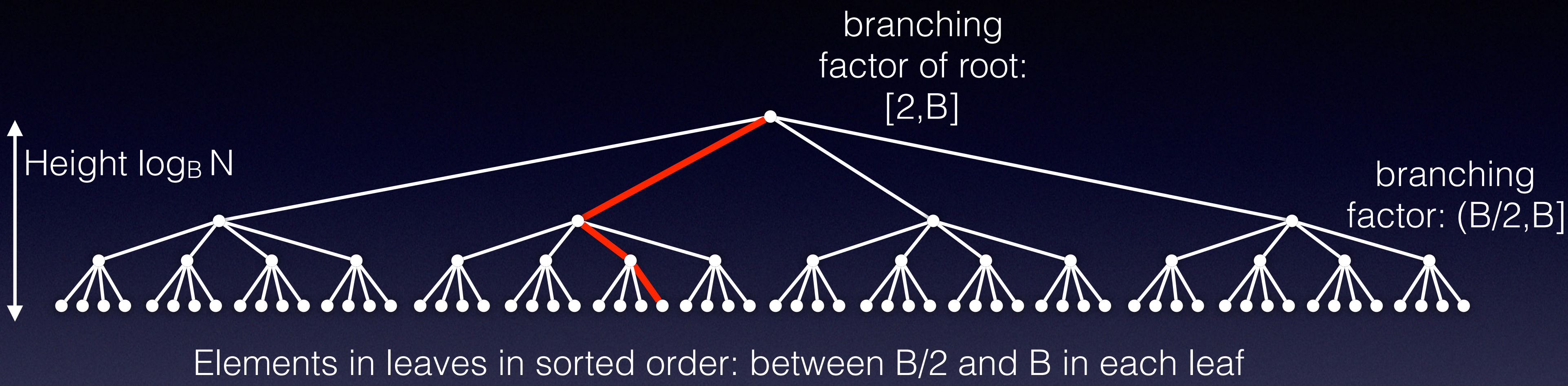


Elements in leaves in sorted order: between B/2 and B in each leaf

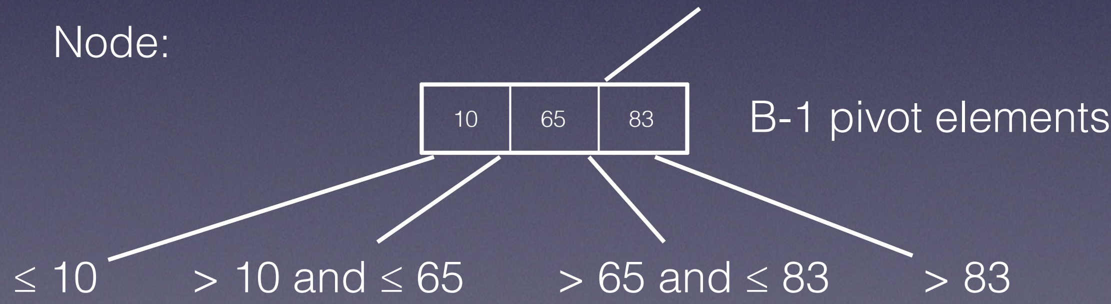
- Operations
  - insert(k, v)
  - delete(k)
  - v = search(k)
  - $[v_1, v_2, \ldots] = \text{range-query}(k_1, k_2)$



B-trees



- A node/leaf can be stored in O(1) blocks.
- Search uses $O(\log_B N)$ I/Os.



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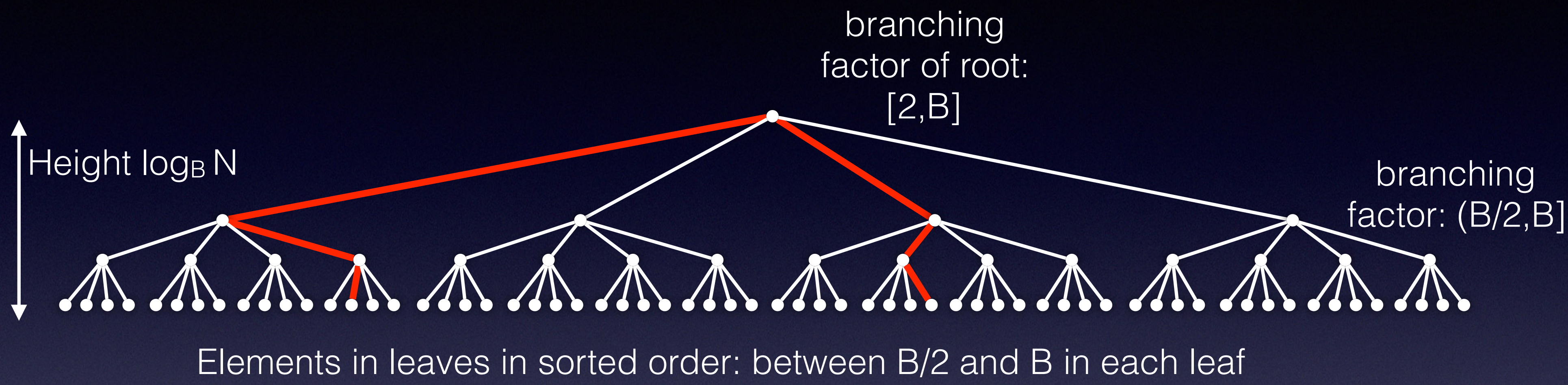
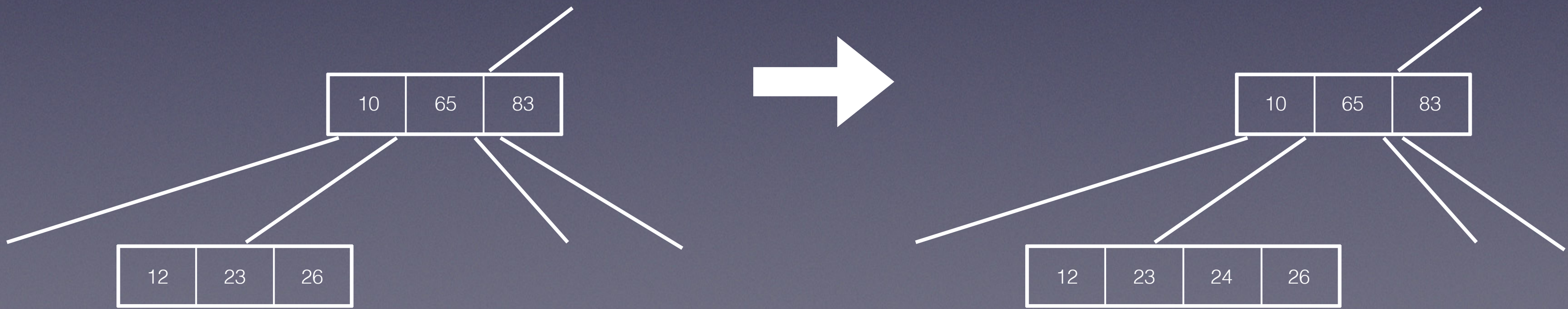


- Range query $[q_1,q_2]$:
  - search down T for q_1 and q_2 (or successor and predecessor).
  - report the elements in the leaves between the leaves containing (successor of) q_1 and (predecessor of) q_2.
- #I/Os = O(log_B N + occ/B).

Insertion in B-tree

- **Insert(k, v)**
  - search for relevant leaf u and insert (k,v) in u.
  - If u now contains B+1 elements:
    - split it into two leaves u′ and u′′.
    - update parent(u)
    - If parent(u) now has degree B+1 recursively split it.
  - If root split: add a new root node with degree 2 (height of tree grows)

- **Example**. B= 5. Insert(24, v)



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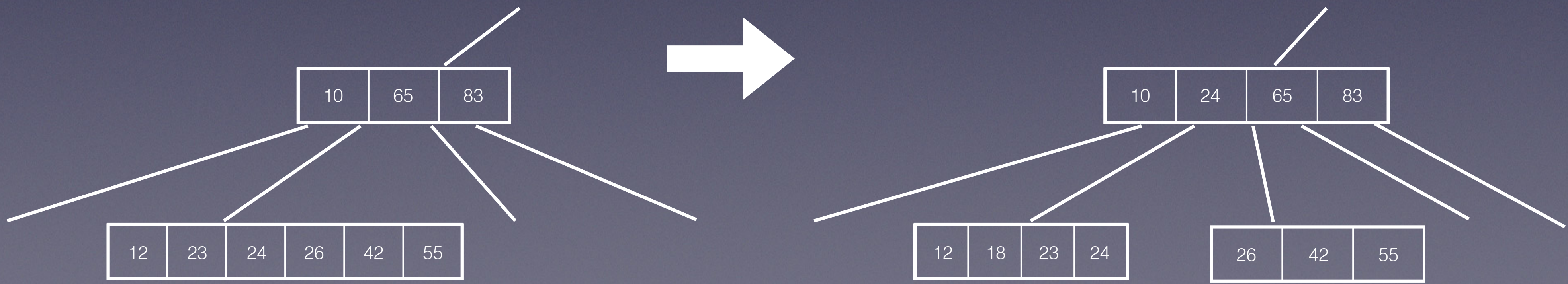
- **Example**. B= 6. Insert(18, v)



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- #I/Os = $O(\log_B N)$

Deletion in B-tree

- **Delete(k)**
  - search for relevant leaf u and delete element with key k in u.
  - If u now contains B/2 - 1 elements:
    - merge u with its sibling u′. If this results in u containing more than B elements split it into two leaves.
    - update parent(u)
    - If parent(u) now has degree B/2 - 1 recursively merge it.
  - If root has degree 1: delete root (height decreases)

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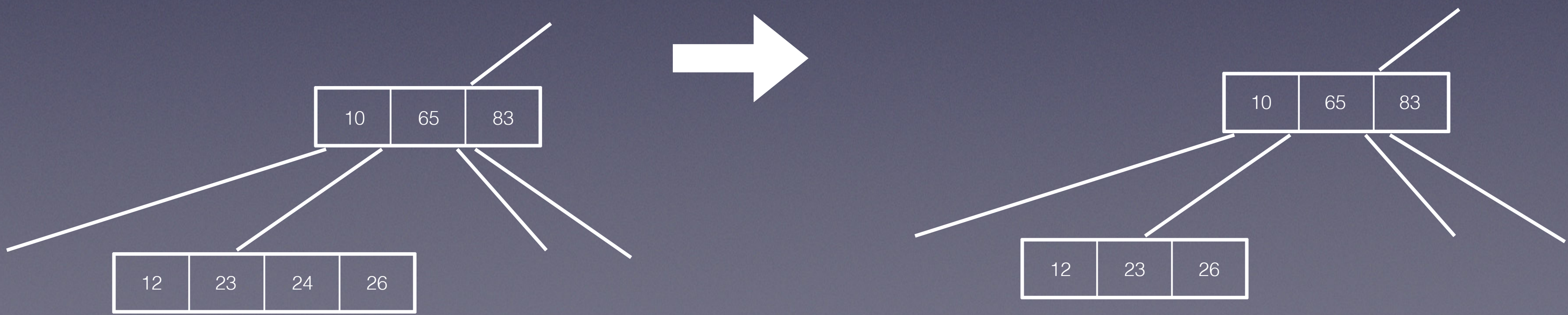

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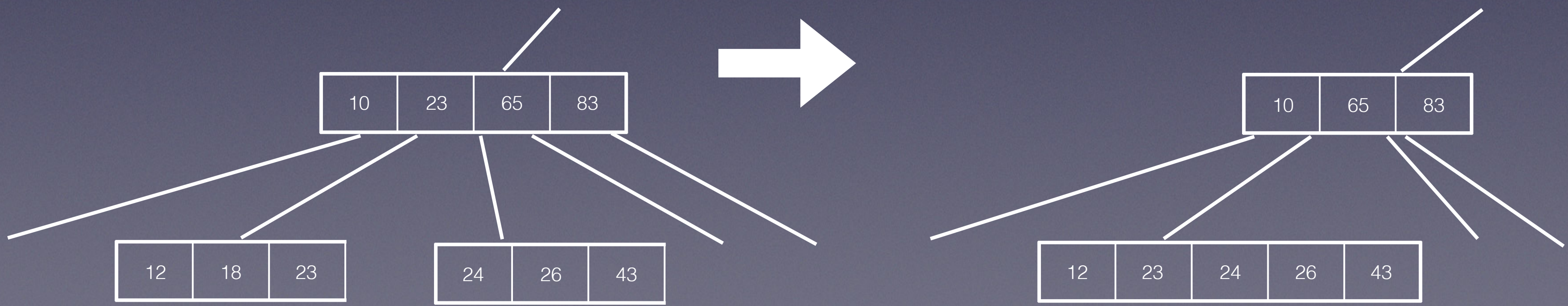

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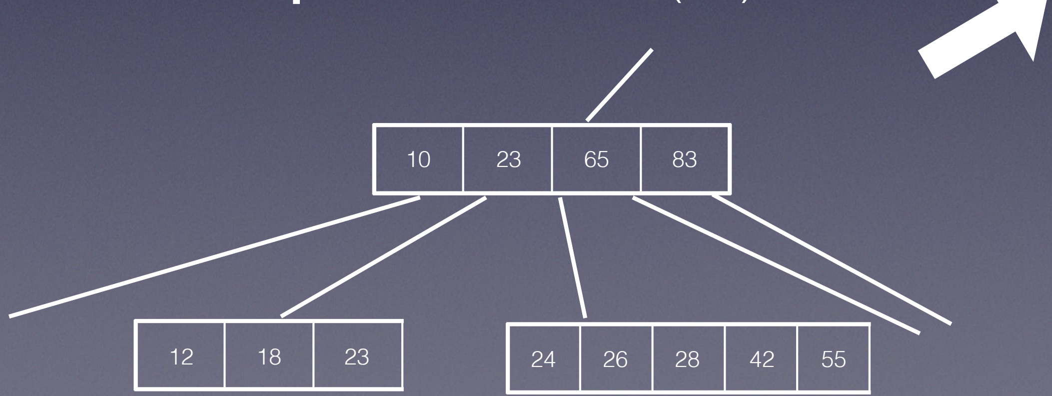

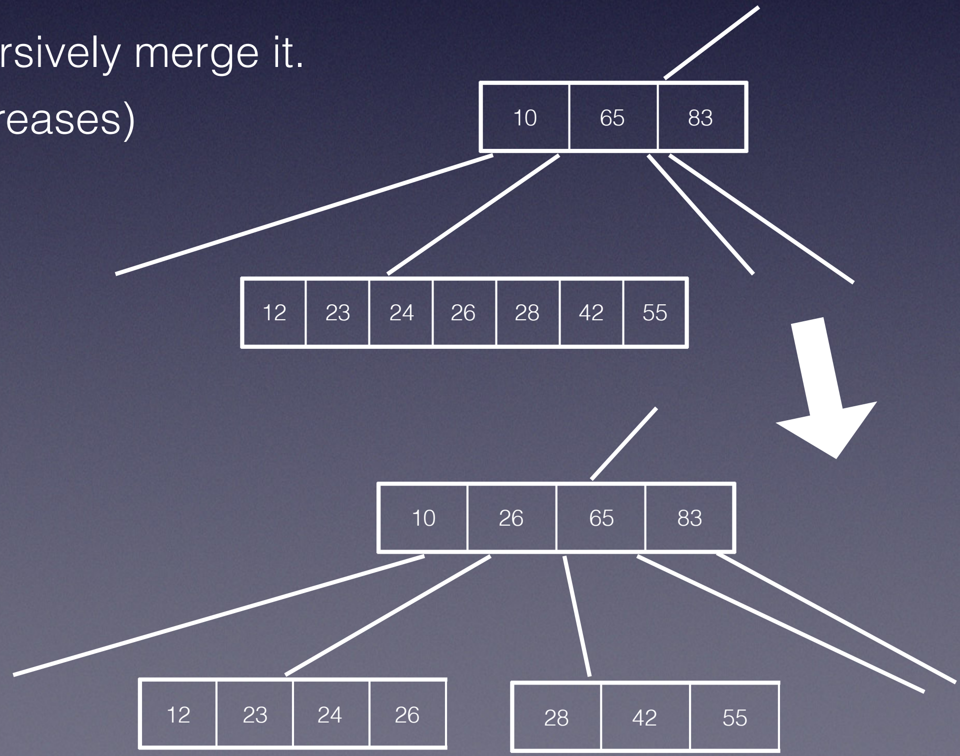

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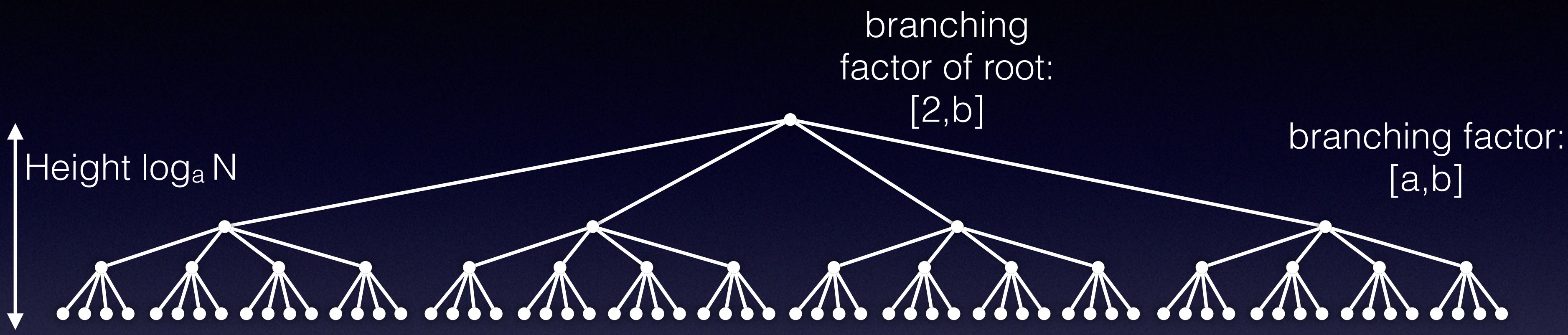
  - #I/Os = $O(\log_B N)$

(a,b)-trees



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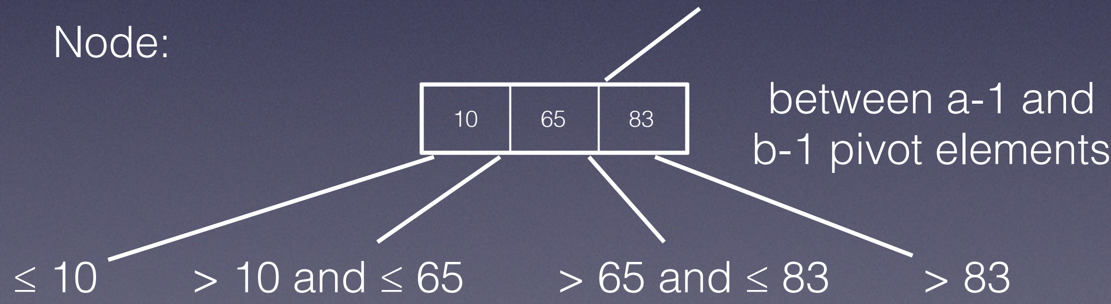
Node:

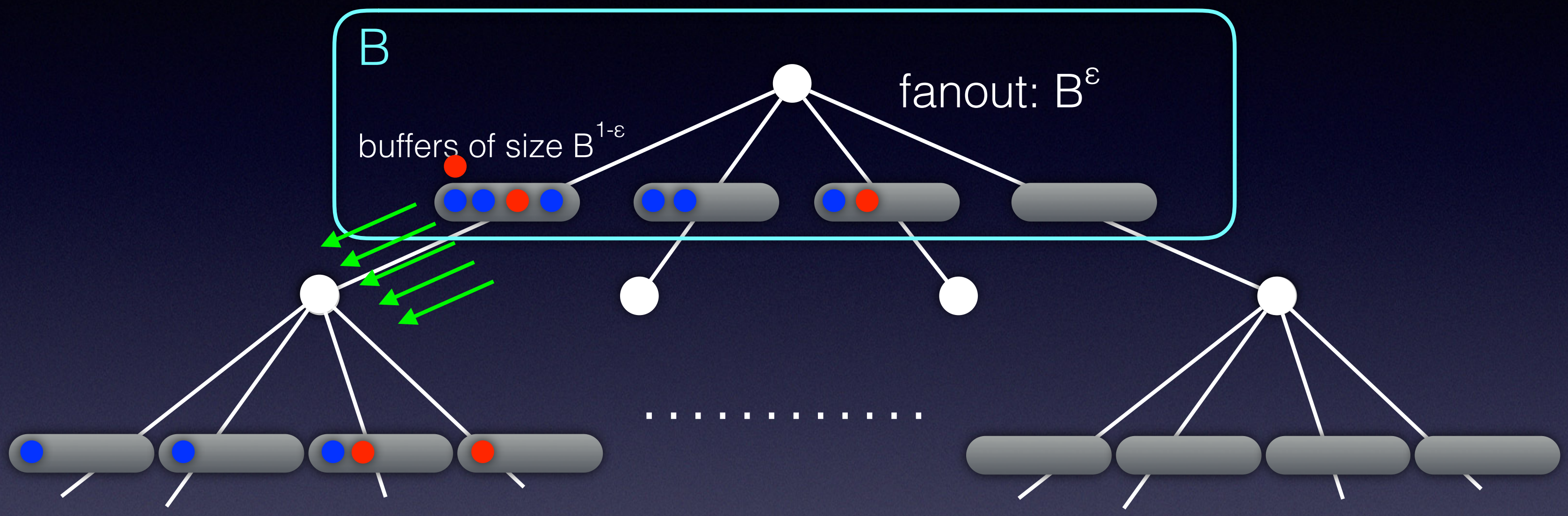


a B-tree is an (a,b)-tree with $a,b=\Theta(B)$

Amortized updates in (a,b)-trees

- If $b \geq 2a$ then the number of rebalancing operations caused by an update $O(1/a)$ amortized

B^{ε} trees



- For $0 \leq \varepsilon \leq 1$:
  - Updates: $O((\log_{1+B^{\varepsilon}} N)/B^{1-\varepsilon})$
  - Point query: $O(\log_{1+B^{\varepsilon}} N))$
  - Range query:
    - $O((\log_{1+B^{\varepsilon}} N)+ occ/B)$

- $\varepsilon = 1/2$:
  - Updates: $O((\log_B N)/\sqrt{B})$
  - Point query: $O(\log_B N)$
  - Range query:
    - $O((\log_B N)+ occ/B)$