

# Mandatory Exercise: Hashing

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**1 Bipartite 2-Paths** Let  $G = (V_1 \cup V_2, E)$  be a bipartite graph, with  $|V_1| + |V_2| = n$  nodes and  $m \geq n$  edges. We want a data structure for  $G$  that supports the following operation on any pair of nodes  $v$  and  $u$  from  $V_1$ .

- $\text{2-path}(v, u)$ : Return yes if there is a path of length 2 between  $v$  and  $u$  and no otherwise.

Solve the following exercises.

- 1.1** Give a data structure that uses  $O(|V_1|^2)$  space and supports fast 2-path queries. The query time should be  $o(m)$ .
- 1.2** Give a data structure that uses  $O(m)$  space and supports fast 2-path queries. *Hint*: A good solution has a query time that depends on the degrees of the input nodes.

You do not need to consider preprocessing time.