

Mandatory Exercise: Hashing

Philip Bille

Inge Li Gørtz

1 The Great Baking Competition DTU is having the annual baking competition. For the competition there are n cake recipes that each contains a set of ingredients needed for that specific recipe. There are m different ingredients numbered from 1 to m , and for each recipe i we have a list L_i containing the set of ingredients needed in recipe i . The participants in the competition are going to bake cakes two and two in parallel. The two participants in a round must bake different cakes. Two cakes cannot be baked in parallel if they contain the same ingredient. Therefore, the planning committee needs to solve the following problem. Given the lists of ingredients L_1, L_2, \dots, L_n they would like a data structure that can answer the following query

- `bake-in-parallel(i, j)`: return yes if cake i and cake j can be baked in parallel, i.e., if they do not contain a common ingredient, and no otherwise.

The total size of the lists is N , i.e., $\sum_{i=1}^n |L_i| = N$. Solve the following exercises.

- 1.1** Give a data structure that uses $O(n^2)$ space and supports fast `bake-in-parallel` queries. The query time for a `bake-in-parallel` query should be $o(m)$, i.e., asymptotically faster than m .
- 1.2** Give a data structure that uses $O(N)$ space and supports fast queries. Hint: A good solution has a query time that depends on the size of one of the input lists to the `bake-in-parallel` query.