Weekplan: Dynamic Graphs I

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References and Reading

[1] S. Even and Y. Shiloach: *An On-Line Edge Deletion Problem* 1981.

We recommend reading the specified article in detail.

Exercises

The following exercises relate to the concepts of graph algorithms and dynamic graphs.

1 Space travel

1.1 You are commanding the space ship Traveller stuck in a distant part of the galaxy. On board, you have n members of the crew. You now want to divide this crew into four shift teams, $1, 2, 3, 4$, that work on the space ship at disjoint times of the day. Challenge: you want to avoid putting a person on the same team as one of their arch enemies. Each team should be of at least 1 person and at most $n - 3$ persons, and n is greater than 4.

Luckily, due to their advanced star fleet training, each member of your crew is in a conflict with at most 3 different people of the star ship at the same time.

- Show how to model this as a graph problem, and,
- Give an efficient algorithm for solving it.
- What is the running time of your algorithm?

1.2 Now, as time goes by, relationships change, enemies become friends, and vice versa. Still, at all points of time, no person is in a conflict with more than 3 people.

- Show how to model this as a dynamic graph.
- How fast can you perform the necessary update when a conflict disappears?
- How fast can you perform the necessary update when a new conflict arises?
- Is your solution better than assigning people to new teams all over again?

2 Zombie Virus

2.1 Imagine that patient zero has contracted the Zombie Virus, and that the virus is transmitted via handshakes. Assume further that every person in the entire world has a very predictable behaviour: every day, a person meets the same other persons, and they shake hands. Assume furthermore that it takes one night from being infected until being able to infect others.

- Model this as a graph problem:
- In terms of the network, how many days does it take for some person x to get infected?

2.2 Now, suppose people in the network may switch behaviour, and by extensive handwashing, social distancing, and garlic, become unable to contract or pass on the Zombie Virus.

- Model this as a dynamic graph.
- How may this affect a person who is not patient zero?

2.3 Give an efficient algorithm for maintaining the handshake-distance from patient zero to every person in the world, as people dynamically switch behaviour from socially interacting to socially distancing (they never switch back after having switched to distancing). What is the total update time, as a function of (a) the number of persons and (b) the number of original handshake-relationships?

3 This app is new in traffic You are writing the app GetThere which facilitates that people can lookup their shortest route to some destination by car.

- Show how to model this as a graph problem, and,
- Sketch solutions:
  - If you run an algorithm upon each query, what is its running time?
  - If you build a data structure containing answers to all possible queries, what is its space consumption?

However, as the morning progresses, due to accidents and congestion, the time necessary to travel along a given road segment can change significantly, and some road segments can become completely non-traversable.

- Show how to model this as a dynamic graph problem.
- What are the updates to the graph?

Do not attempt to solve it, only model it.
Now, imagine that all the users of your app only want to get either *to DTU* or *home from DTU*.

- Would this simplify your task?
- What would be simpler, the updates to the graph, or the questions asked by the app users?

The following exercises relate a bit more to the specific article we read.

4 Black box data structure.

4.1 Assume that you are given as a black box a fully-dynamic data structure A which is able to maintain a single source shortest paths tree in weighted (non-negative) directed graphs and to answer queries on the distance from the source to any other vertex of the graph.

Present an algorithm that given a weighted directed graph $G = (V, E)$ computes its distance matrix only by using the data structure A for some graph G'.

Efficiency: Make G' as small (close to the size of G) as possible, and make few updates and queries.

4.2 Now, assume instead you are given a data structure B which is only decremental. Present an algorithm that given a weighted directed graph $G = (V, E)$ computes its distance matrix only by using the data structure A for some graph G' with almost the same number of vertices and edges as G.

5 Decremental reachability in acyclic graphs. Let $G = (V, E)$ be a directed acyclic graph and let $v \in V$. A reachability tree of v is a tree that contains paths to all the vertices that are reachable from v in G. Present a data structure for maintaining a decremental reachability tree from v whose total update time is $O(|E|)$.

6 Decremental reachability in general graphs. Let $G = (V, E)$ be a directed graph (that might contain cycles). Present a data structure for maintaining a decremental reachability tree from $v \in V$ whose total update time is $O(|E||V|)$.

Acknowledgement Thanks to Liam Roditty for inspiration to the last three exercises.