Suffix Trees

- String Indexing/String Dictionaries
- Tries
- Suffix Trees

Inge Li Gørtz

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String Indexing

- String indexing problem. Let S be a string of characters from alphabet Σ. Preprocess S into data structure to support:
  - search(P): Return the starting positions of all occurrences of P in S.
- Example.
  - S = yabbadabbado
  - search(abba) = {1,6}
- String dictionary problem. Let S = $\{S_1, S_2, \ldots, S_k\}$ be a set of strings of characters from alphabet Σ. Preprocess S into data structure to support:
  - search(P): Return yes if P = S_i for some S_i in S.
  - prefix-search(P): Return all strings in S for which P is a prefix.

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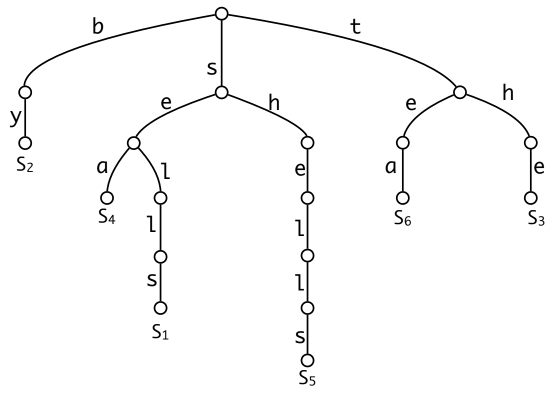
Tries

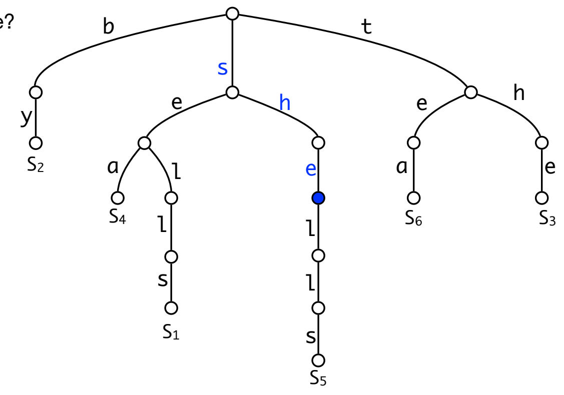
- Text retrieval



- Trie over the strings: sells, by, the, sea, shells, tea.

Tries

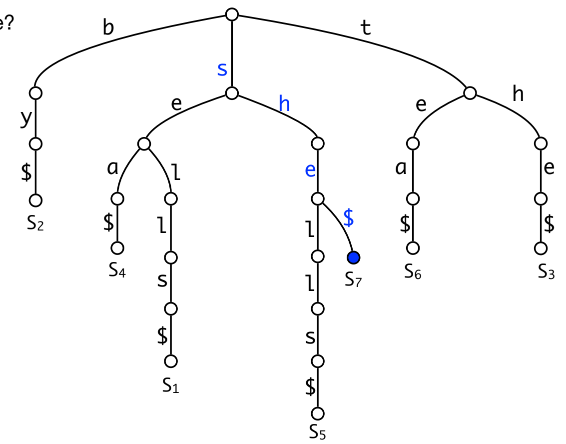
- Text retrieval
- Prefix-free?



- Trie over the strings: sells, by, the, sea, shells, tea, she.

Tries

- Text retrieval
- Prefix-free?



- Trie over the strings: sells$, by$, the$, sea$, shells$, tea$, she$.

Tries

- Fredkin 1960]. Retrieval. Store a set of strings in a rooted tree such that:
  - Each edge is labeled by a character. Edges to children of a node are sorted from left-to-right alphabetically.
  - Each root-to-leaf path represents a string in the set. (obtained by concatenating the labels of edges on the path).
  - Common prefixes share same path maximally.
- Properties of the trie. A trie T storing a collection S of s strings of total length *n* from an alphabet of size *d* has the following properties:
  - How many children can a node have?
  - How many leaves does T have?
  - What is the height of T?
  - What is the number of nodes in T?

Tries

- Fredkin 1960]. Retrieval. Store a set of strings in a rooted tree such that:
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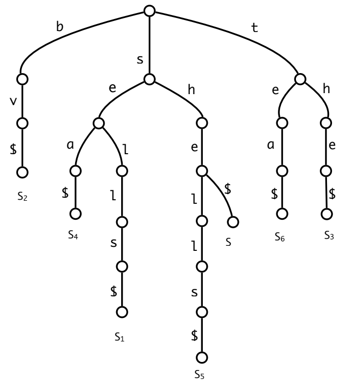
  - Common prefixes share same path maximally.
- Properties of the trie. A trie T storing a collection S of s strings of total length *n* from an alphabet of size *d* has the following properties:
  - How many children can a node have? at most d
  - How many leaves does T have? s
  - What is the height of T? length of longest string
  - What is the number of nodes in T? O(n)

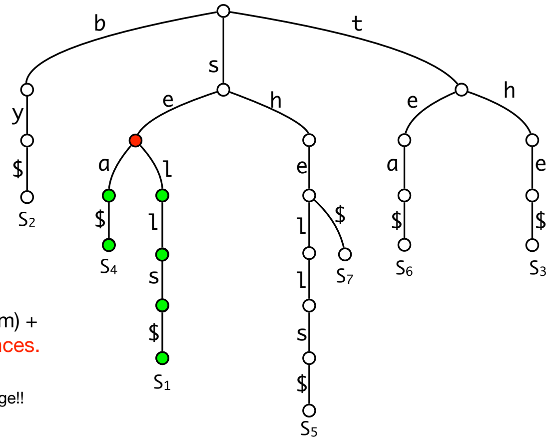
Trie

- Search time: O(d) in each node => O(dm).
  - O(m) if d constant.
  - d not constant: use dictionary
    - Perfect hashing O(1)
    - Balanced BST: O(log d)
- Time and space for a trie (for constant d):
  - O(m) for searching for a string of length m.
  - O(n) space.
  - Preprocessing: O(n)



Tries

- Prefix search: return all words in the trie starting with “se”



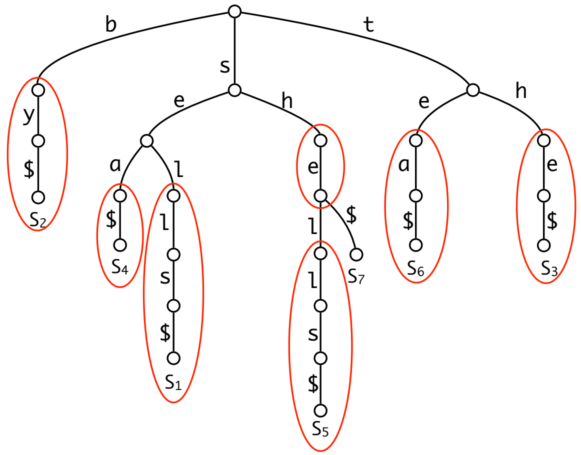
- Time for prefix search: O(m) + time to report all occurrences.

Could be large!!

- Solution: compact tries.

Tries

- Compact trie. Chains of nodes with a single child is merged into a single edge.



Tries

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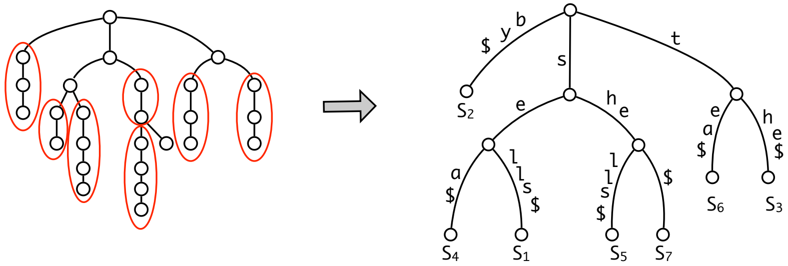

- Properties of the compact trie. A compact trie T storing a collection S of s strings of total length *n* from an alphabet of size *d* has the following properties:
  - Every internal node of T has at least 2 and at most *d* children.
  - T has *s* leaves
  - The number of nodes in T is < 2s.

Trie

- Time and space for a compact trie (constant d).
  - O(m) for searching for a string of length m.
  - O(m + occ) for prefix search, where occ = #occurrences
  - O(n) space.
  - Preprocessing: O(n)

Suffix Trees

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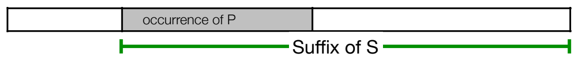
Suffix tree

- String indexing problem. Given a string S of characters from an alphabet Σ. Preprocess S into a data structure to support
  - Search(P): Return starting position of all occurrences of P in S.

- Observation: An occurrence of P is a prefix of a suffix of S.



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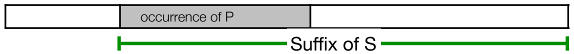
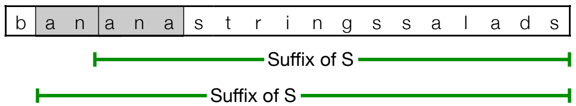
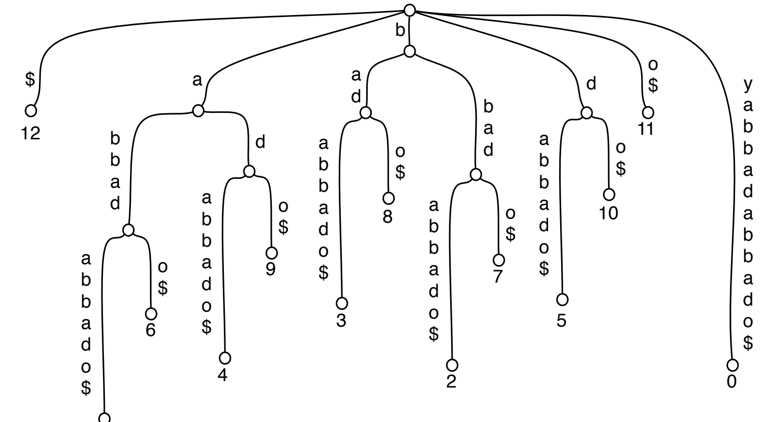


- Example: P = ana.



Suffix Trees

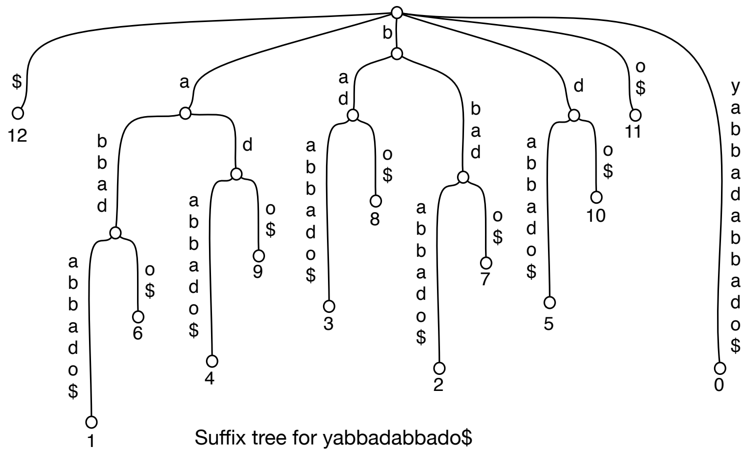
- Suffix trees. The compact trie of all suffixes of S.



Suffix tree for yabbadabbado$

Suffix Trees

- Suffix trees. The compact trie of all suffixes of S.
- Search for abba:



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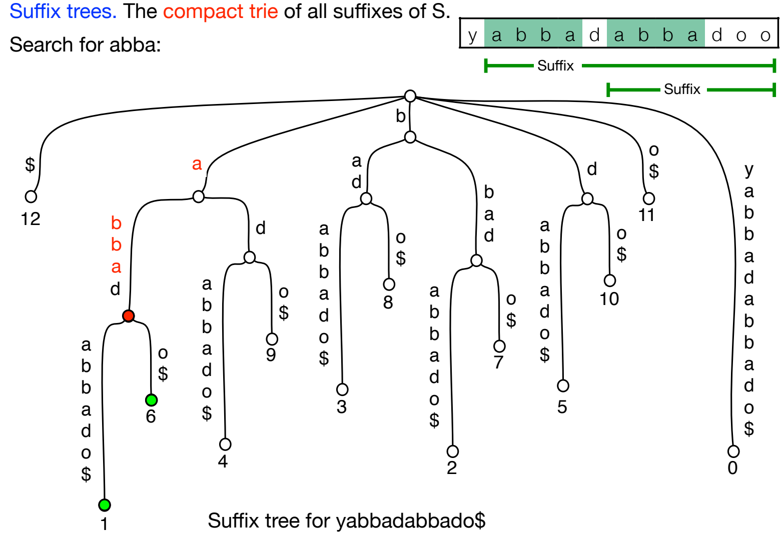
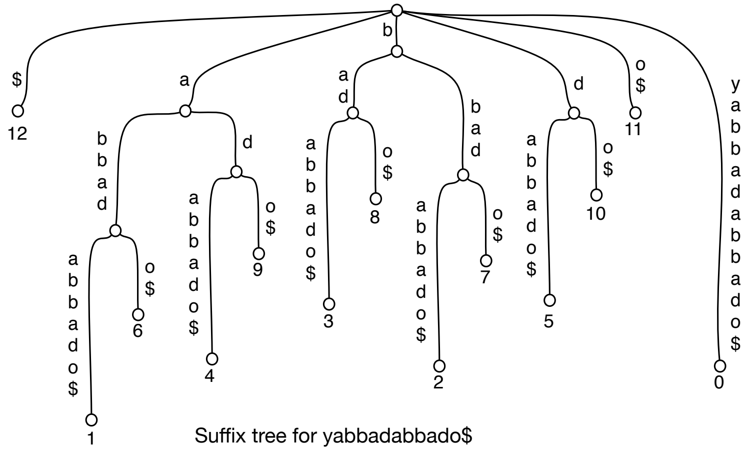


Suffix tree for yabbadabbado$

Suffix Trees

- Suffix trees. The compact trie of all suffixes of S.
- Space?

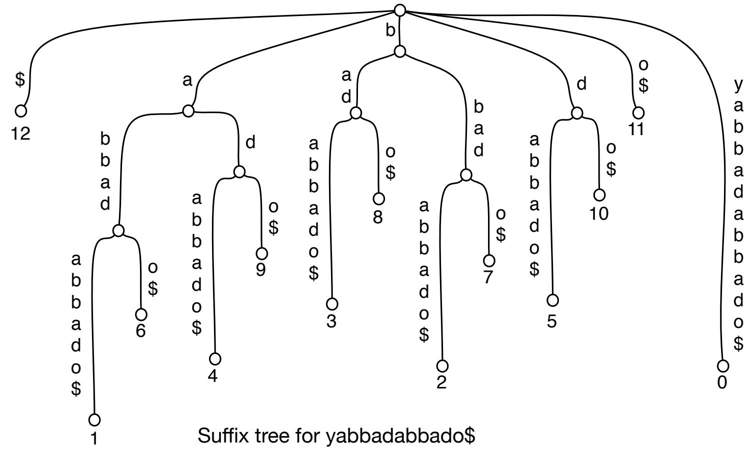


Suffix tree for yabbadabbado$

Suffix Trees

- Suffix trees. The compact trie of all suffixes of S.
- Store S and store edge labels by reference to S.

0	1	2	3	4	5	6	7	8	9	10	11	12
y	a	b	b	a	d	a	b	b	a	d	o	$



Suffix tree for yabbadabbado$

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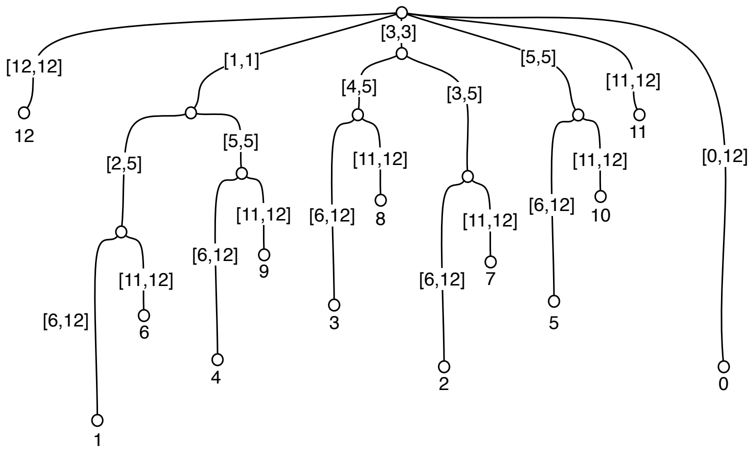

Suffix Trees

- Space.
  - Number of edges + space for edge labels + string
  - ⟹ O(n) space
- Preprocessing. O(sort(n,|Σ|))
- sort(n,|Σ|) = time to sort n characters from an alphabet Σ.
- Search(P): O(m+occ).

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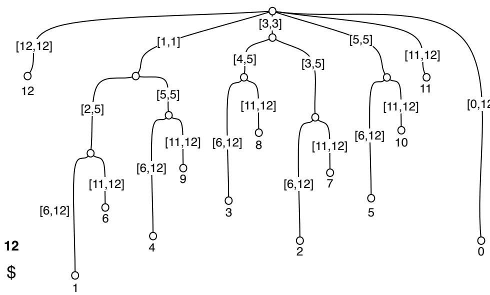


Suffix tree for yabbadabbado$

Suffix Trees

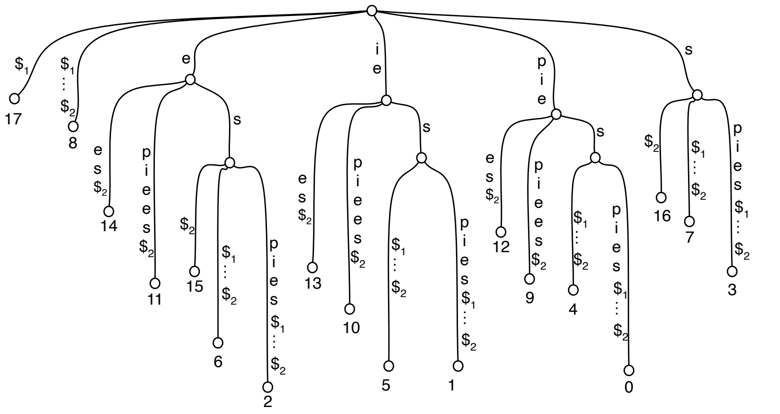
- Theorem. We can solve the string indexing problem in
  - O(n) space and sort(n,|Σ|) preprocessing time.
  - O(m + occ) time for queries.

Suffix Trees

- Applications.
  - Approximate string matching problems
  - Compression schemes (Lempel-Ziv family, ...)
  - Repetitive string problems (palindromes, tandem repeats, ...)
  - Information retrieval problems (document retrieval, top-k retrieval, ...)
  - ...

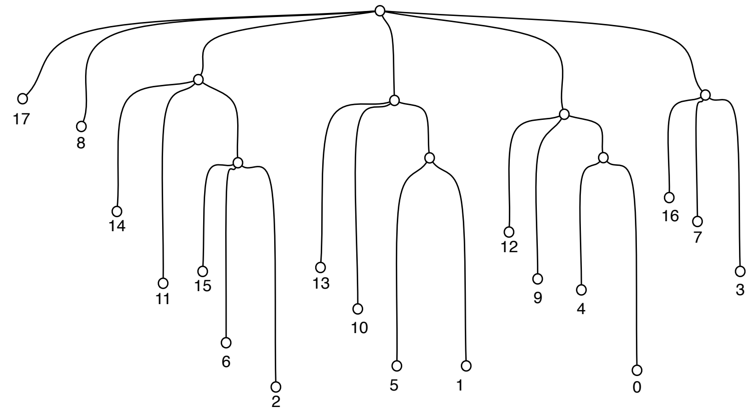
Longest common substring

- Find longest common substring of strings S_1 and S_2.
- Construct the suffix tree over $S_1\$_1S_2\$_2$.
- Example. Find longest common substring of piespies and piepiees:
  - Construct suffix tree of piespies$\$_1$piepiees$\$_2$.



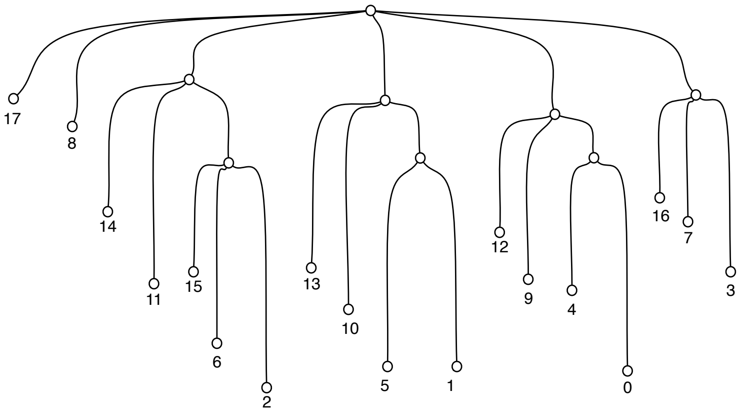
Longest common substring

- Suffix tree of piespies$\$_1$piepiees$\$_2$.



Longest common substring

- Suffix tree of piespies$_1$piepiees$_2$.
- Mark leaves: ■ = S_1 ▲ = S_2



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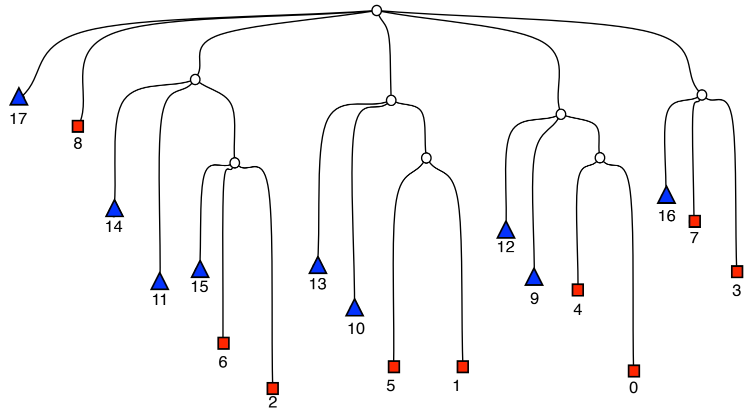

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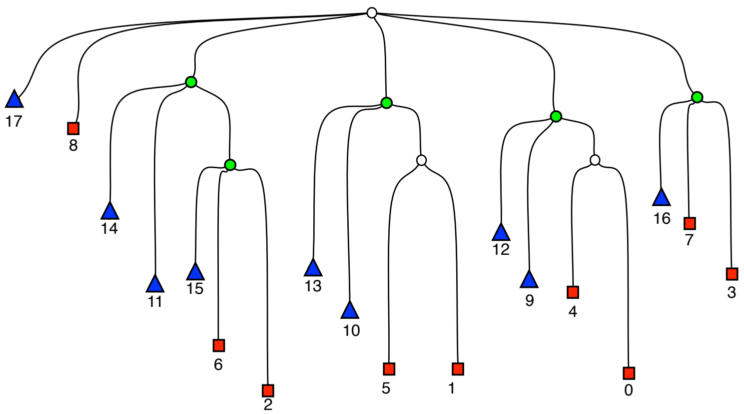
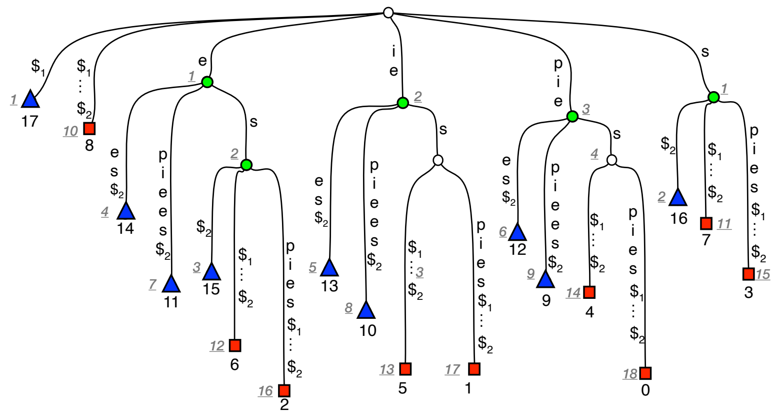


Longest common substring

- Suffix tree of piespies$_1$piepiees$_2$.
- Mark leaves: ■ = S_1 ▲ = S_2
- Add string depth.



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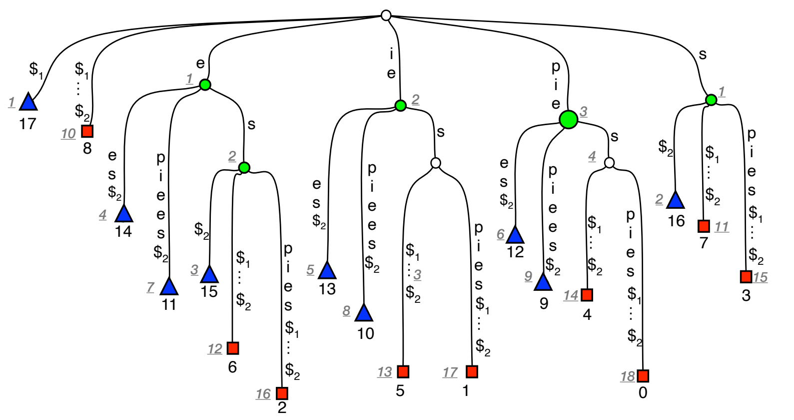


Longest common substring

- Using a suffix tree we can solve the longest common substring problem in linear time (for a constant size alphabets).

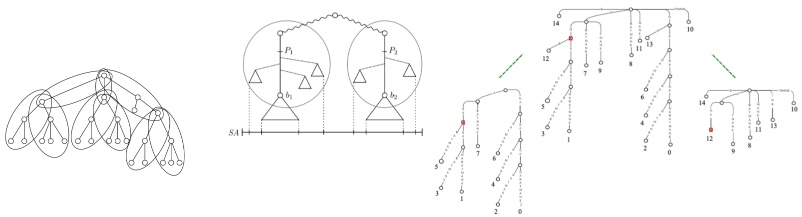
Gapped Pattern Indexing

- Search for name:

"Inge Gørtz" or "Inge Li Gørtz" ➡ "Inge" [1-4] "Gørtz"

Gapped pattern

- Techniques:
  - *Clustering* of the suffix tree
  - *Induced suffix tree decomposition:* recursively divide the suffix tree in half by index in the string.



Gapped Pattern Matching

- Search for my name:

"Inge Gørtz" or "Inge Li Gørtz" ➡ "Inge" [1-4] "Gørtz"

Gapped pattern

- General gapped pattern:

$$P_1\ [a_1, b_1]\ P_2\ [a_2, b_2]\ P_3\ \cdots\ [a_{k-1}, b_{k-1}]\ P_k$$

- Other variants (indexing):
  - Wildcards
  - Top-k closest consecutive occurrences