# External Memory II

- Searching with Fast Updates
- Searching Strings

Philip Bille

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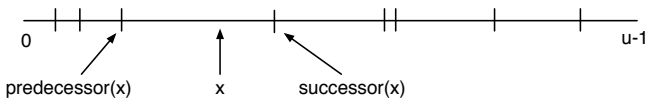
## Searching

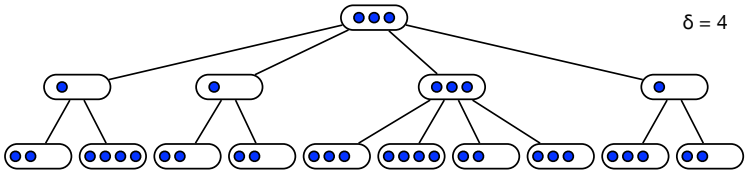
- Searching. Maintain a set $S \subseteq U = \{0, ..., u-1\}$ supporting
  - member(x): determine if $x \in S$
  - predecessor(x): return largest element in $S \leq x$.
  - successor(x): return smallest element in $S \geq x$.
  - insert(x): set $S = S \cup \{x\}$
  - delete(x): set $S = S - \{x\}$




## Searching

- Applications.
  - Relational data bases.
  - File systems.

## B-tree

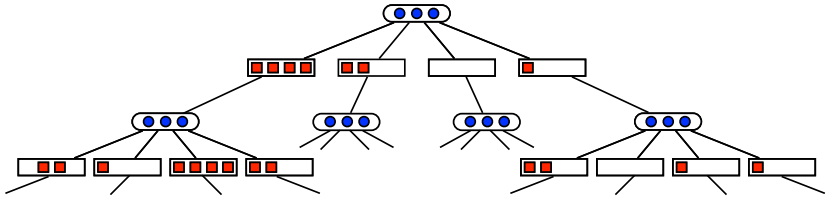



- B-tree of order δ = Θ(B) with N keys.
  - Keys in leaves. Routing elements in internal nodes.
  - Degree between δ/2 and δ.
  - Root degree between 2 and δ.
  - Leaves store between δ/2 and δ keys.
  - All leaves have the same depth.
- Height. $\Theta(\log_\delta (N/B)) = \Theta(\log_B N)$
- Search and update. $O(\log_B N)$ I/Os.

## $B^\varepsilon$-tree

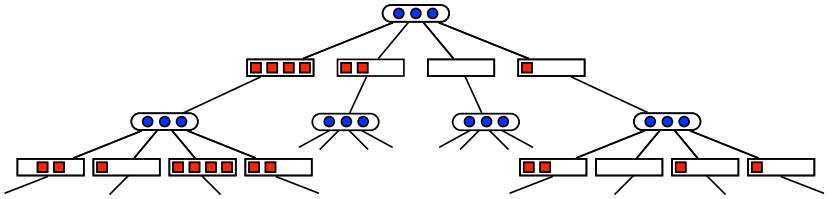
- Idea.
  - Speed up updates by buffering them at each node along the path to a leaf.
  - Move many updates together in each I/O.
  - Search (almost) as before.
  - $\varepsilon \in (0, 1]$ is a parameter.
- Solution in 2 steps.
  - Focus on $\sqrt{B}$-tree ($\varepsilon = 1/2$).
    - Searching in $O(\log_B N)$ I/Os.
    - Updates in $O((\log_B N)/\sqrt{B})$ amortized.
  - Generalize to any $\varepsilon$.

## $\sqrt{B}$-tree



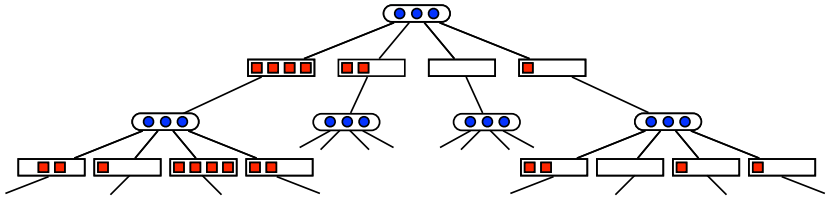
- $\sqrt{B}$-tree with N keys.
  - B-tree of degree $\Theta(\sqrt{B})$ with buffers of size $\Theta(\sqrt{B})$ at each edge.
  - Buffer stores delayed updates in subtree.
  - Nodes and child buffers stored together in O(1) blocks.
- Height. $\Theta(\log_{\sqrt{B}} N) = \Theta(\log_B N)$

## $\sqrt{B}$-tree



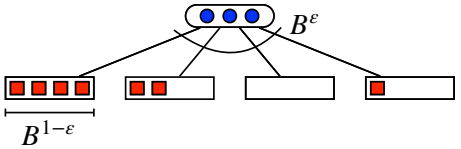
- Searching.
  - Find leaf using routing elements. Check buffers along path.
- I/Os. $O(\log_B N)$.

## $\sqrt{B}$-tree



- Updates.
  - Insert update message into buffer at child.
  - If buffer full, flush and recurse at child.
  - If we reach leaf, rebalance tree as B-tree.
- I/Os. Intuition: A flush moves $\sqrt{B}$ messages ⇒ O((log$_B$ N)/$\sqrt{B}$) amortized I/Os.
  - Assign (clog$_B$ N)/$\sqrt{B}$ credits to each update for constant c > 1.
  - Put c/$\sqrt{B}$ credits each node on path.
  - ⇒ We can pay for buffer overflows and rebalancing.

## $B^\varepsilon$-tree




- $B^\varepsilon$-tree with N keys.
  - B-tree of degree $\Theta(B^\varepsilon)$ with buffers of size $\Theta(B^{1-\varepsilon})$ at each edge.
- Searching. $O\left(\frac{\log_B N}{\varepsilon}\right)$ I/Os.
- Updates. $O\left(\frac{\log_B N}{\varepsilon B^\varepsilon}\right)$ I/Os.

## $B^\varepsilon$-tree

| | Search | Update |
|---|---|---|
| B-tree | $O(\log_B N)$ | $O(\log_B N)$ |
| $\sqrt{B}$-tree | $O(\log_B N)$ | $O\left(\frac{\log_B N}{\sqrt{B}}\right)$ |
| $B^\varepsilon$-tree | $O\left(\frac{\log_B N}{\varepsilon}\right)$ | $O\left(\frac{\log_B N}{\varepsilon B^\varepsilon}\right)$ |

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## String Searching

- String searching. Maintain a set S = {$S_1$, $S_2$, ..., $S_K$} of K strings of total length N supporting the following operations:
  - search(P): return string in S with longest common prefix with P.
  - insert(P): set S = S ∪ {P}
  - delete(P): set S = S - {P}

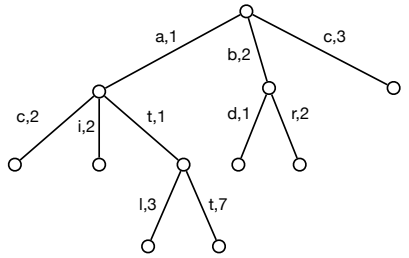
## String Searching

- Which solutions do we know (on the RAM model)?

## String Searching

- Goal.
  - Searching in $O(\log_B N + |P|/B)$ I/Os.
  - Ignore insert and delete.
- Solution in 3 steps.
  - Blind tries.
  - String B-trees.
  - String B-trees with fast searches.

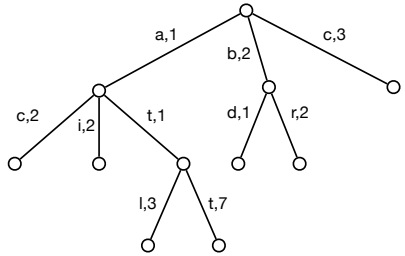
## Blind Trie




1. ace
2. aid
3. atlas
4. atom
5. attenuate
6. bid
7. bird
8. car

- Data structure.
  - Sorted set of strings.
  - Compact trie for S. Edges store first char + string length.
- Space.
  - Strings: O(N)
  - Trie: O(K)
  - ⇒ O(N)

## Blind Trie

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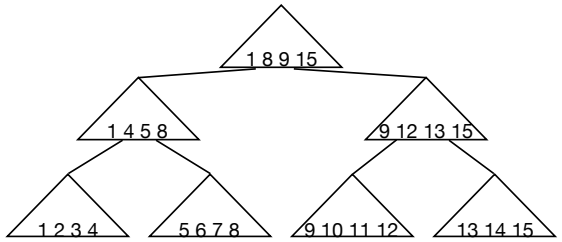
- Search. Traverse and verify candidate.
- Time. O(|P|)

## String B-tree

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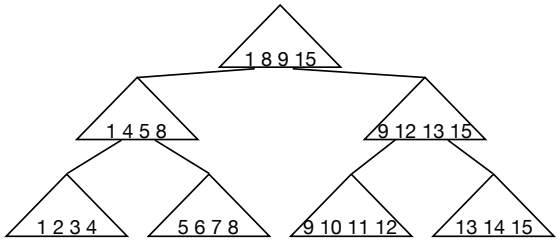
9. cod
10. dog
11. fit
12. lid
13. patent
14. sun
15. zoo

- Data structure. Combination of B-tree and blind tries.
  - Sorted set of strings.
  - Nodes store blind trie over B strings.
  - Leftmost and rightmost string in subtree stored for each child.
- Space.
  - Strings: O(N)
  - B-tree: O(B) per node.
  - ⇒ O(N).

## String B-tree

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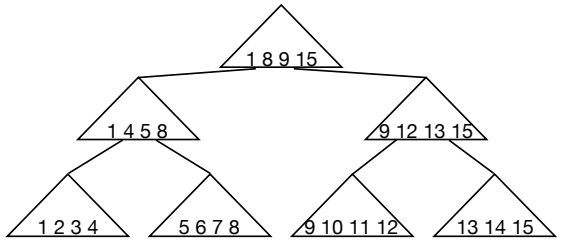
- Searching.
  - Find leaf using routing with blind tries.
  - Verify leaf.
- I/Os.
  - Routing at node: O(|P|/B) I/Os.
  - ⇒ O((|P|/B)$\log_B$ N) I/Os.

## String B-tree

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- Fast searching.
  - Remember longest prefix at each node.
  - Verify leaf.
- I/Os.
  - Routing in total: O(|P|/B) I/Os.
  - ⇒ O((|P|/B) + $\log_B$ N) I/Os.

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