Weekplan: Distributed Algorithms II

Philip Bille Inge Li Gørtz Eva Rotenberg

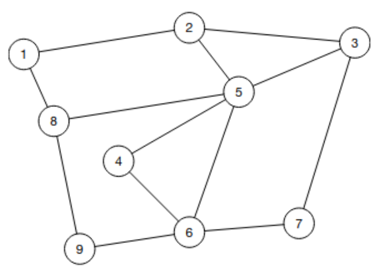
References and Reading

[1] Jukka Suomela: Distributed algorithms (latest version)
chapter 5 (and the introductory part of chapter 4).

We recommend reading chapter 5 of [1] in detail.

Exercises

1 (w) Run the BFS tree algorithm on the graph below. Let $t(v)$ be the round in which $a(v)$ was set to 1. For each node maintain $d(v)$, $C(v)$, $a(v)$, and $t(v)$. Indicate $p(v)$ by marking the edge from v to $p(v)$. Assume that a node u with $d(u) = \perp$ always accept the proposal from the node with the smallest identifier.



2 Edge counting (ex.5.2 from [1]) Assume the graph is connected.

Give an efficient algorithm for counting the number of edges in the graph: When the algorithm terminates, each vertex should output the number of edges in the graph.

(Reminder: Give an algorithm means describe it, argue for correctness, and analyse its running time, i.e. number of rounds.)

Analyse the running time as a function of the diameter of the graph.

3 Detecting bipartiteness (ex.5.3 from [1]) Assume the graph is connected.

Give an efficient algorithm for detecting whether the graph is bi-partite, i.e, the vertices can be divided into two groups A and B such that each edge is of the form ab with $a \in A$ and $b \in B$.

When the algorithm terminates, each vertex should output "yes" if it is bipartite and "no" otherwise..

Analyse the running time as a function of the diameter of the graph.

4 Detecting completeness (ex.5.4 from [1]) Assume the graph is connected.

Give an efficient algorithm for detecting whether the graph is complete, i.e, there is an edge between every pair of vertices.

When the algorithm terminates, each vertex should output "yes" if it is complete and "no" otherwise..

Can you do this in only a constant number of rounds?

5 Maximal independent set

An independent set is *maximal* if no vertex can be added to it without violating its independence.

5.1 Give an example of a 3-colouring of a path where no colour class is a maximal independent set.

5.2 Assume you are given a 4-colouring of an n-vertex graph with maximum degree 3.

Can you transform this into a maximal independent set?

Can you do this in a constant number of rounds?

5.3 Does your solution above generalise to the situation where you are given a $\Delta + 1$ colouring of a Δ-degree graph? What is the running time?

"Star-exercises" Solve star-exercises from the notes, from sections 5 and 6.