Weekplan: Distributed Algorithms II

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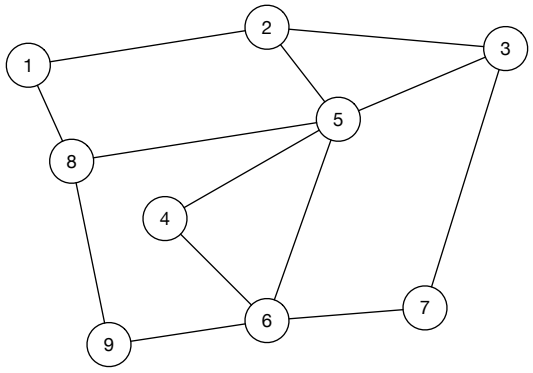
References and Reading

[1] Distributed Algorithms Chapter 5. By Jukka Suomela.

Exercises

1 $[w]$ **BFS tree** Run the BFS tree algorithm on the graph below. Let $t(v)$ be the round in which $a(v)$ was set to 1. For each node maintain $d(v)$, $C(v)$, $a(v)$, and $t(v)$. Indicate $p(v)$ by marking the edge from v to $p(v)$. Assume that a node u with $d(u) = \perp$ always accept the proposal from the node with the smallest identifier.



2 Edge counting (Ex. 5.2 from [1]) The *edge counting problem* is defined as follows: each node has to output the value $|E|$, i.e., it has to indicate how many edges there are in the graph. Assume that the input graph is connected. Design an algorithm that solves the edge counting problem in the CONGEST model in time $O(\text{diam(G)})$.

3 Detecting bipartite graphs (Ex. 5.3 from [1]) Assume that the input graph is connected. Design an algorithm that solves the following problem in the CONGEST model in time $O(\text{diam(G)})$:

- If the input graph is bipartite, all nodes output 1.
- Otherwise all nodes output 0.

4 Detecting complete graphs (Ex. 5.4 from [1]) . Assume that the input graph is connected. Design an algorithm that solves the following problem in the CONGEST model in time O(1):

- If the input graph is a complete graph, all nodes output 1.
- Otherwise all nodes output 0.

5 MST Design an algorithm that computes a minimum spanning tree in $O(n \log n)$ rounds in the CONGEST model. Assume the following:

- Each edge weight is an integer polynomial in n, allowing it to be sent as a single message.
- Each node u stores a list of pairs (w, v) where w is the weight of edge (u, v) for each neigbor v of u.

Hint: Maintain a spanning forest and recursively merge connected components.

6 Gathering (Ex. 5.5 from [1]) Assume that the input graph is connected. Sections 4.2 and 5.2 describe how to gather full information on the input graph in time $O(\mathrm{diam}(G))$ in the LOCAL model (no message size limit).

Design an algorithm that solves the problem in time $O(|E|)$ in the CONGEST model.