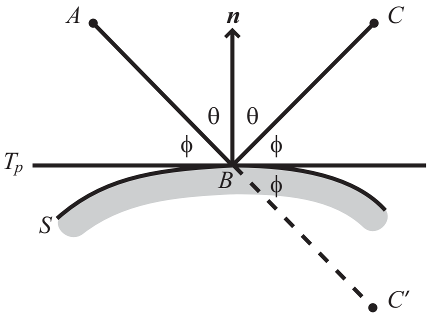
Reflected rays

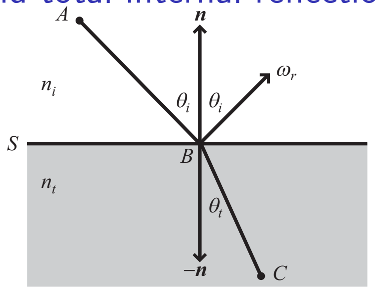
plane of incidence: containing B, spanned by $\boldsymbol{n}$ and $A - B$



$$
\begin{aligned}
\vec{\omega}_i &= (A - B)/\|A - B\| \\
\vec{n} &= \boldsymbol{n}/\|\boldsymbol{n}\| \\
\cos\theta_i &= \vec{\omega}_i \cdot \vec{n} \\
\vec{\omega}_r &= (C - B)/\|C - B\| \\
&= 2\cos\theta_i \vec{n} - \vec{\omega}_i \\
&= 2(\vec{\omega}_i \cdot \vec{n})\vec{n} - \vec{\omega}_i
\end{aligned}
$$

- Light path from A to C by reflection off a mirror surface S.
- Applying Hero's principle:
  Choose a point B on S such that the distance $\overline{AB} + \overline{BC}$ is minimal.
- Let C' be the mirror image of C in the tangent plane T_p at B.
  Then $\overline{AB} + \overline{BC} = \overline{AB} + \overline{BC'}$ is minimal if $\overline{AC'}$ is straight.
- This requires that the vector $C' - B$ is in the plane of incidence.
- **Law of reflection**. The reflected ray lies in the plane of incidence; the angle of reflection equals the angle of incidence.

Refraction and total internal reflection



$$\cos\theta_i = \vec{\omega}_i \cdot \vec{n}$$
$$\sin^2\theta_i = 1 - (\vec{\omega}_i \cdot \vec{n})^2$$
$$\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i$$
$$\cos^2\theta_t = 1 - \left(\frac{n_i}{n_t}\right)^2 (1 - (\vec{\omega}_i \cdot \vec{n})^2)$$
$$\vec{t} = \frac{\cos\theta_i \vec{n} - \vec{\omega}_i}{\sin\theta_i}$$
$$\vec{t}\sin\theta_t = \frac{n_i}{n_t}((\vec{\omega}_i \cdot \vec{n})\vec{n} - \vec{\omega}_i)$$

- In the plane of incidence:
  - $\vec{\omega}_i = (A - B)/\|A - B\|$ is the direction of incidence,
  - $\vec{t} = \boldsymbol{t}/\|\boldsymbol{t}\|$ is the unit length tangent of S at B,
  - $\vec{n} = \boldsymbol{n}/\|\boldsymbol{n}\|$ is the unit length normal of S at B,
  - $\vec{\omega}_t = \vec{t}\sin\theta_t - \vec{n}\cos\theta_t$ is the direction of the refracted ray.
- We have total internal reflection if $\cos^2\theta_t < 0$ (all is reflected, no refracted ray, $n_i > n_t$).
- Otherwise: $\vec{\omega}_t = \frac{n_i}{n_t}((\vec{\omega}_i \cdot \vec{n})\vec{n} - \vec{\omega}_i) - \vec{n}\sqrt{1 - \left(\frac{n_i}{n_t}\right)^2 (1 - (\vec{\omega}_i \cdot \vec{n})^2)}$