02941 Physically Based Rendering

Sun and Sky and Colour and Environment Maps

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Dynamic range

- Ambient luminance levels for some common lighting environments:

Condition	**Illumination (cd/m^2)**
Starlight	10^{-3}
Moonlight	10^{-1}
Indoor lighting	10^{2}
Sunlight	10^{5}
Maximum intensity of common monitors	10^{2}

Reference

- Reinhard, E., Ward, G., Pattanaik, S., Debevec, P., Heidrich, W., and Myszkowski, K. High Dynamic Range Imaging: Acquisition, Display and Image-Based Lighting, second edition, Morgan Kaufmann/Elsevier, 2010.

High dynamic range imaging

- Why doesn't the camera see what I see?
  - The camera has a much smaller dynamic range (several orders of magnitude measured in cd/m^2).
  - The part of the visible dynamic range captured by the camera is determined by the size of the aperture and the exposure time.
- Exposure is usually changed in stops.
  - A stop is a power-of-two exposure step (halving the exposure time while keeping the aperture constant will decrease the exposure by 1 stop, for example).
- High dynamic range imaging:
  - Keep the camera still and take images at multiple exposures.
  - Combine several low dynamic range images into one high dynamic range image (HDR image capture).
  - Map the high dynamic range image to a low dynamic range display (tone reproduction).
- HDRI was once Hollywood's best kept secret [Bloch 2007].

References

- Bloch, C. *The HDRI Handbook: High Dynamic Range Imaging for Photographers and CG Artists*. Rocky Nook, 2007.
- Reinhard, E., Ward, G., Pattanaik, S., Debevec, P., Heidrich, W., and Myszkowski, K. *High Dynamic Range Imaging: Acquisition, Display and Image-Based Lighting*, second edition, Morgan Kaufmann/Elsevier, 2010.

HDR image capture

- Exposure time from 30 s to 1 ms in 1-stop steps.

- Combining to get high dynamic range:

$$L_{ij} = \sum_{k=1}^{N} \frac{f^{-1}(Z_{ijk})w(Z_{ijk})}{\Delta t_k} \Big/ \sum_{k=1}^{N} w(Z_{ijk}) \ ,$$

where Z_{ijk} is response-weighted radiant exposure of pixel ij in capture k of exposure time Δt_k, w is a weighting function (to deal with overexposure), and f is the camera response function.

References

- Debevec, P. E., and Malik, J. Recovering high dynamic range radiance maps from photographs. In *Proceedings of ACM SIGGRAPH 97*, pp. 369–378, August 1997.
- Reinhard, E., Ward, G., Pattanaik, S., Debevec, P., Heidrich, W., and Myszkowski, K. *High Dynamic Range Imaging: Acquisition, Display and Image-Based Lighting*, second edition, Morgan Kaufmann/Elsevier, 2010.

Tone reproduction

left Linear mapping of all dynamic range.

middle Linear mapping of lower 0.1% of dynamic range.

right Histogram adjustment [Ward et al. 1997].

References

- Debevec, P. E., and Malik, J. Recovering high dynamic range radiance maps from photographs. In *Proceedings of ACM SIGGRAPH 97*, pp. 369–378, August 1997.
- Ward, G., Rushmeier, H., and Piatko, C. A visibility matching tone reproduction operator for high dynamic range scenes. *IEEE Transactions on Visualization and Computer Graphics 3*(4), pp. 291–306, 1997.

RGBE encoding (the .hdr format)

- RGBE $\rightarrow$ RGBA

FIGURE 3.2 Bit breakdown for the 32-bit/pixel RGBE (and XYZE) encodings.

$$
\begin{aligned}
R_W &= \frac{R_M + 0.5}{256} 2^{E-128} \\
G_W &= \frac{G_M + 0.5}{256} 2^{E-128} \\
B_W &= \frac{B_M + 0.5}{256} 2^{E-128}
\end{aligned}
$$

References

- Reinhard, E., Ward, G., Pattanaik, S., Debevec, P., Heidrich, W., and Myszkowski, K. *High Dynamic Range Imaging: Acquisition, Display and Image-Based Lighting*, second edition, Morgan Kaufmann/Elsevier, 2010.

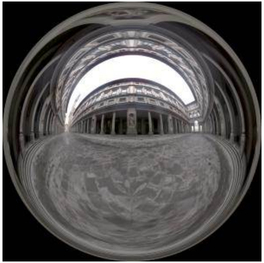
Light probes

- The angular map

$$r = \frac{\arccos(-D_z)}{2\pi\sqrt{D_x^2 + D_y^2}}$$

$$(u, v) = \left(\frac{1}{2} + rD_x, \frac{1}{2} + rD_y\right) ,$$

where (D_x, D_y, D_z) is the look-up direction into the environment map.



References

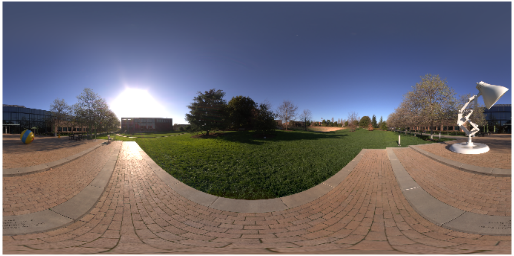
- Debevec, P. Image-based lighting. *IEEE Computer Graphics and Applications 22*(2), pp. 26-34, 2002.

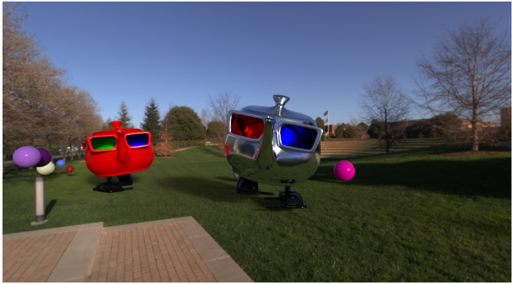
Panoramic Format

- The latitude-longitude map

$$
\begin{aligned}
u &= \frac{1}{2} + \frac{1}{2\pi} \arctan\left(\frac{D_x}{-D_z}\right) \\
v &= \frac{1}{\pi} \arccos(-D_y) \quad ,
\end{aligned}
$$

where (D_x, D_y, D_z) is the look-up direction into the environment map.





References

- Reinhard, E., Ward, G., Pattanaik, S., Debevec, P., Heidrich, W., and Myszkowski, K. *High Dynamic Range Imaging: Acquisition, Display and Image-Based Lighting*, second edition, Morgan Kaufmann/Elsevier, 2010.
- Pixar RenderMan Holdout Workflow: `https://renderman.pixar.com/resources/RenderMan_20/risHoldOut.html`.

Environment illumination

$$L_r(\mathbf{x},\vec{\omega}) = \int_{2\pi} f_r(\mathbf{x},\vec{\omega}_i,\vec{\omega})L_i(\mathbf{x},\vec{\omega}_i)\cos\theta\,\mathrm{d}\omega_i$$

$$\approx \frac{\rho_d(\mathbf{x})}{\pi}\sum_{j=1}^{N} V(\vec{\omega}_j)L_{\text{env}}(\vec{\omega}_j)\cos\theta\,\Delta\omega_j \quad ,$$

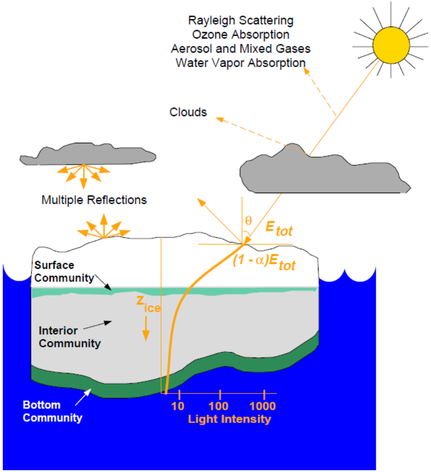
- $L_{\text{env}}(\vec{\omega}_j)$ is the radiance received from an environment map by look-up using $\vec{\omega}_j$.
- To cast shadows on the environment, one can use the concept of holdouts: inserting geometry to model objects seen in the environment.
- Holdout shading:

$$L_N(\mathbf{x},\vec{\omega}) = L_{\text{env}}(\vec{\omega})\frac{1}{N}\sum_{j=1}^{N} V(\vec{\omega}_j) \quad ,$$

N is number of samples or light sources.

The colour of the sky

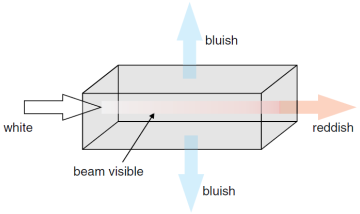
The atmosphere

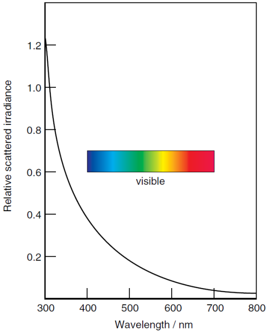


Reference

- Belém, A. L. Modeling Physical and Biological Processes in Antarctic Sea Ice. PhD Thesis, Fachbereich Biologie/Chemie der Universität Bremen, February 2002.

Rayleigh scattering





- Quote from Lord Rayleigh [On the light from the sky, its polarization and colour. *Philosophical Magazine 41*, pp. 107–120, 274–279, 1871]:

If I represent the intensity of the primary light after traversing a thickness x of the turbid medium, we have

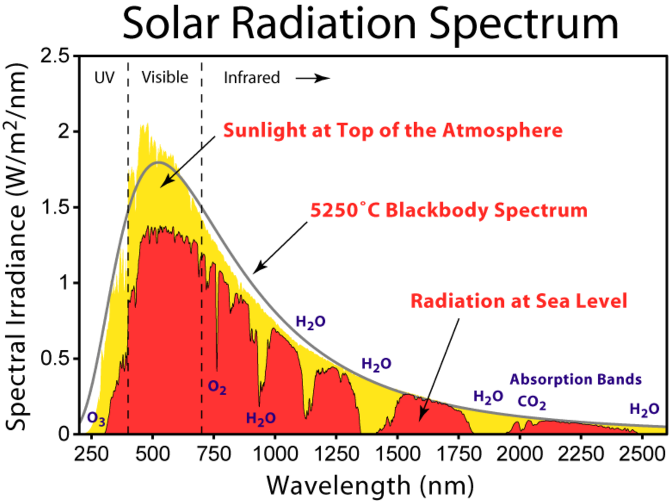
$$dI = -kI\lambda^{-4}\,dx\,,$$

where k is a constant independent of λ*. On integration,*

$$I = I_0 e^{-k\lambda^{-4}x}\,,$$

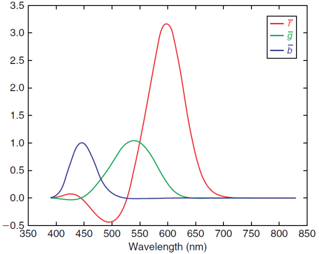
if I_0 *correspond to* $x = 0$, *—a law altogether similar to that of absorption, and showing how the light tends to become yellow and finally red as the thickness of the medium increases.*

Solar radiation

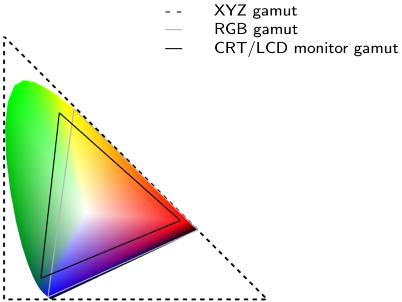


[Source: https://en.wikipedia.org/wiki/Sunlight]

Colorimetry



CIE color matching functions



The chromaticity diagram

$$
\begin{aligned}
R &= \int_{\mathscr{V}} C(\lambda)\bar{r}(\lambda)\,\mathrm{d}\lambda \\
G &= \int_{\mathscr{V}} C(\lambda)\bar{g}(\lambda)\,\mathrm{d}\lambda \\
B &= \int_{\mathscr{V}} C(\lambda)\bar{b}(\lambda)\,\mathrm{d}\lambda \ ,
\end{aligned}
$$

where $\mathscr{V}$ is the interval of visible wavelengths and $C(\lambda)$ is the spectrum that we want to transform to RGB.

Gamut mapping

- Gamut mapping is mapping one tristimulus color space to another.
- Gamut mapping is a linear transformation. Example:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} .$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.2405 & -1.5371 & -0.4985 \\ -0.9693 & 1.8760 & 0.0416 \\ 0.0556 & -0.2040 & 1.0572 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- Y in the XYZ color space is called *luminance*.
- Luminance is a measure of how bright a scene appears.
- From the linear transformation above, we have

$$Y = 0.2126\,R + 0.7152\,G + 0.0722\,B\,.$$

Tone mapping

- Simplistic tone mapping: scale and gamma correct:

$$(R', G', B') = \left((sR)^{1/\gamma}, (sG)^{1/\gamma}, (sB)^{1/\gamma}\right) .$$

  where s and γ are user-defined parameters.
- The framework uses this:
  - s is 0.03 for the sun and sky,
  - γ is 1.8 and is applied by pressing '*'.

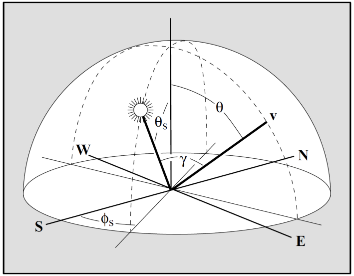
- Another tone mapping operator (Ferschin's exponential mapping):

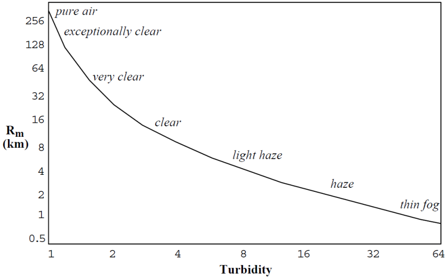
$$(R', G', B') = \left((1 - e^{-R})^{1/\gamma}, (1 - e^{-G})^{1/\gamma}, (1 - e^{-B})^{1/\gamma}\right) .$$

- This is useful for avoiding overexposed pixels.

- Other tone mapping operators use sigmoid functions based on the luminance levels in the scene [Reinhard et al. 2010].

Analytical sky models [Preetham et al. 1999] (input parameters)





- Solar declination angle:

$$\delta = 0.4093 \sin\left(\frac{2\pi(J-81)}{368}\right) .$$

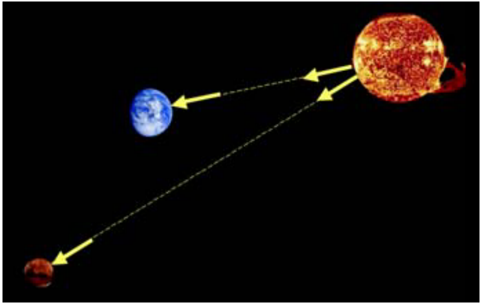
- Solar position:

$$\theta_s = \frac{\pi}{2} - \arcsin\left(\sin\ell \sin\delta - \cos\ell \cos\delta \cos\frac{\pi t}{12}\right) ,$$

$$\phi_s = \operatorname{atan2}\left(-\cos\delta \sin\frac{\pi t}{12}, \cos\ell \sin\delta - \sin\ell \cos\delta \cos\frac{\pi t}{12}\right) ,$$

where $J \in [1, 366]$ is the ordinal day number, t is the solar time, and ℓ is the latitude.

Direct sunlight



- Assume the Sun is a diffuse emitter of total power $3.91 \cdot 10^{26}$ W and the surface area is $6.07 \cdot 10^{18}$ m^2.
- Calculate the radiance from the Sun to Earth.
- Assume the Sun is in zenith and the distance from Sun to Earth is $1.5 \cdot 10^{11}$ m.
- Find the solid angle subtended by the Sun as seen from Earth.
- How much energy is received on a 1×1 cm^2 patch on Earth?
- Note that the solid angle enables us to go from radiance to irradiance. The solar irradiance is useful for specifying a directional light resembling the Sun.

Exercises

- Use a sky model for background colour.
- Implement the link between sky appearance and
  - location on Earth (latitude),
  - day of year and time of day (ordinal day and solar time),
  - and orientation (scene angle with South).
- Use the model to set the RGB power of a directional light resembling the sun.
- Render a sequence of images where the bunny is on a green plane with the sun rising in front of it and setting behind it.
- Load a panoramic texture and use it as environment map. Use the sun model and implement a holdout shader to insert an object in the photographed environment.