02941 Physically Based Rendering

Microfacet Models

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From smooth to rough surfaces

- Triangles can represent any surface.
  - Smooth: Use interpolated vertex normals.
  - Rough: Use triangle face normals.

- If surface features are very small:
  - Triangles are very small (numerical problems).
  - Triangles are numerous (computation is expensive).

- Microfacet BRDF models.
  - A BRDF can replace tiny surface features with randomly oriented microfacets.

Mesoscopic Bidirectional Reflectance Distribution Function

- In general, we have the bidirectional scattering-surface reflectance distribution function (BSSRDF):

$$S(X; \boldsymbol{x}_i, \vec{\omega}_i; \boldsymbol{x}_o, \vec{\omega}_o) = \frac{\mathrm{d}L_o(\boldsymbol{x}_o, \vec{\omega}_o)}{\mathrm{d}\Phi_i(\boldsymbol{x}_i, \vec{\omega}_i)}\,,$$

  where X is the object boundary and L_o is the radiance reflected in the direction $\vec{\omega}_o$ at the position $\boldsymbol{x}_o$ due to the flux incident at the position $\boldsymbol{x}_i$ from the direction $\vec{\omega}_i$.
- Suppose we consider surface scattering only, or interior scattering so large that an element of irradiance ($\mathrm{d}E = L_i \cos\theta_i \mathrm{d}\omega_i$) has an influence only for $\boldsymbol{x}_o \in A_i$, then

$$\int_{A_i} S(X; \boldsymbol{x}_i, \vec{\omega}_i; \boldsymbol{x}_o, \vec{\omega}_o)\,\mathrm{d}A_i \to \frac{\mathrm{d}L_o(\boldsymbol{x}_o, \vec{\omega}_o)}{\mathrm{d}\Phi_i(\boldsymbol{x}_i, \vec{\omega}_i)}\,\mathrm{d}A = \frac{\mathrm{d}L_o(\boldsymbol{x}, \vec{\omega}_o)}{\mathrm{d}E(\boldsymbol{x}, \vec{\omega}_i)} = f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o)$$

  for $A_i \to \mathrm{d}A_o$ such that $\boldsymbol{x}_i \approx \boldsymbol{x}_o = \boldsymbol{x}$.
- This is a special case of the BRDF (f_r) for which we may assume $\mathrm{d}A_i = \mathrm{d}A_o = \mathrm{d}A$.

Microfacet surfaces

- A microfacet surface is modelled by a BRDF that scatters light in more than one direction.
- One way to describe this is by a distribution of microfacet normals.

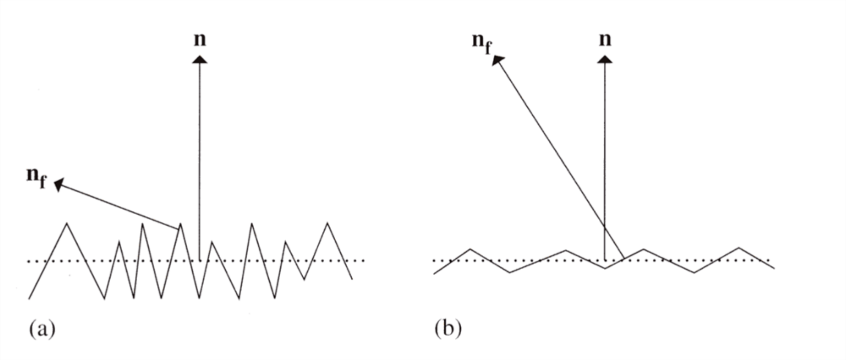

Figure by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

The origins

scalar diffraction by surface elements around a plane

- The scattering of electromagnetic waves from rough surfaces.
  - Overview by Beckmann and Spizzichino [1963].
    - How to develop facet normal distribution functions.

V-grooves

- Translation to geometrical optics.
  - Theory for off-specular reflection from roughened surfaces by Torrance and Sparrow [1967].
    - Introducing the BRDF model

$$f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o) = \frac{FGD}{4(\vec{n} \cdot \vec{\omega}_i)(\vec{n} \cdot \vec{\omega}_o)} = \frac{FGD}{4 \cos \theta_i \cos \theta_o} \ ,$$

where
F is the Fresnel reflectance,
G is the geometrical attenuation factor,
D is the facet normal distribution function.

References

- Beckmann, P., and Spizzichino, A. *The Scattering of Electromagnetic Waves from Rough Surfaces*. International Series of Monographs on Electromagnetic waves, Vol. 4, Pergamon Press, 1963.
- Torrance, K. E., and Sparrow, E. M. Theory of off-specular reflection from roughened surfaces. *Journal of the Optical Society of America 57*(9), pp. 1105-1114, September 1967.

Geometrical attenuation

- Important effects to consider:
  (a) Masking.
  (b) Shadowing.
  (c) Interreflections.

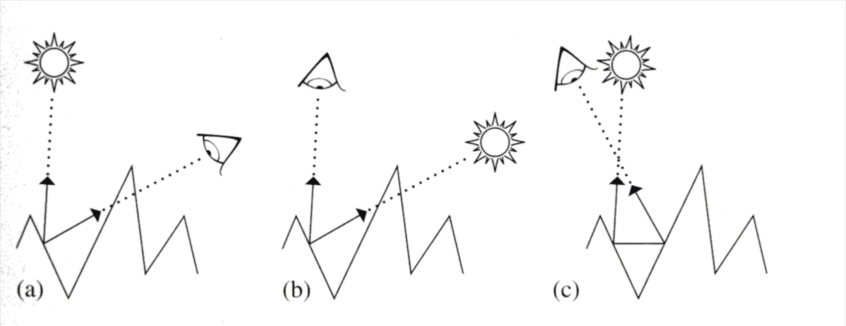

Figure by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

Facet normal distributions

- Assuming perfectly specular microfacets, facets with normals

$$\vec{\omega}_h = \frac{\vec{\omega}_o + \vec{\omega}_i}{\|\vec{\omega}_o + \vec{\omega}_i\|}$$

  contribute to the reflected radiance in the direction $\vec{\omega}_o$.
- Subscripts denote: i - incident, o - observed, h - half.
- $D(\vec{\omega}_h)$ is the facet normal distribution. Then
  - $D(\vec{\omega}_h)\,\mathrm{d}\omega_h$ is the proportion of facets with normals in the solid angle $\mathrm{d}\omega_h$.
  - $D(\vec{\omega}_h)\,\mathrm{d}\omega_h\,\mathrm{d}A$ is the proportion of facets in the area $\mathrm{d}A$ with normals in $\mathrm{d}\omega_h$.
  - $D(\vec{\omega}_h)\,\mathrm{d}\omega_h\,\cos\theta_h \mathrm{d}A$ is the proportion of facets in the projected area $\cos\theta_h \mathrm{d}A$ with normals in $\mathrm{d}\omega_h$, where $\cos\theta_h = \vec{\omega}_i \cdot \vec{\omega}_h$.
- The flux incident at the projected area of the microfacets with normals in $\mathrm{d}\omega_h$ is therefore (using the definition of radiance)

$$\mathrm{d}\Phi_i = L_i D(\vec{\omega}_h)\,\mathrm{d}\omega_h\,\cos\theta_h \mathrm{d}A\,\mathrm{d}\omega_i \quad ,$$

  where L_i is radiance incident from the direction $\vec{\omega}_i$.

Facet normal distributions

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- Taking into account Fresnel reflectance and geometrical attenuation, the reflected flux is

$$\mathrm{d}\Phi_o = F(\cos\theta_h)G(\vec{\omega}_i,\vec{\omega}_o)\,\mathrm{d}\Phi_i \quad .$$

- The BRDF model is then (using $\mathrm{d}E = L_i\cos\theta_i\,\mathrm{d}\omega_i$)

$$\begin{aligned} f_r(\boldsymbol{x},\vec{\omega}_i,\vec{\omega}_o) &= \frac{\mathrm{d}L_o}{\mathrm{d}E} = \frac{\mathrm{d}\Phi_o}{\cos\theta_o\,\mathrm{d}A\,\mathrm{d}\omega_o}\Big/\,\mathrm{d}E \\ &= \frac{F(\cos\theta_h)G(\vec{\omega}_i,\vec{\omega}_o)\,L_i D(\vec{\omega}_h)\,\mathrm{d}\omega_h\;\cos\theta_h \mathrm{d}A\,\mathrm{d}\omega_i}{\cos\theta_o\,\mathrm{d}A\,\mathrm{d}\omega_o\,L_i\cos\theta_i\,\mathrm{d}\omega_i} \quad . \end{aligned}$$

Facet normal distributions

- The BRDF model is then (using $\mathrm{d}E_i = L_i \cos\theta_i \,\mathrm{d}\omega_i$)

$$f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o) = \frac{\mathrm{d}\omega_h}{\mathrm{d}\omega_o} \frac{\cos\theta_h \, F(\cos\theta_h) G(\vec{\omega}_i, \vec{\omega}_o) D(\vec{\omega}_h)}{\cos\theta_i \cos\theta_o} \; .$$

- Expressing the solid angles in spherical coordinates with $\vec{\omega}_i$ as zenith, we have

$$\begin{aligned} \mathrm{d}\omega_h &= \sin\theta_h \,\mathrm{d}\theta_h \,\mathrm{d}\phi_h \; , \\ \mathrm{d}\omega_o &= \sin\theta'_o \,\mathrm{d}\theta'_o \,\mathrm{d}\phi'_o \; . \end{aligned}$$

  where θ'_o is the angle between $\vec{\omega}_i$ and $\vec{\omega}_o$.
- Then according to the law of reflection $\phi'_o = \phi_h$ and $\theta'_o = 2\theta_h$.
- This means that

$$\frac{\mathrm{d}\omega_h}{\mathrm{d}\omega_o} = \frac{\sin\theta_h \,\mathrm{d}\theta_h \,\mathrm{d}\phi_h}{\sin(2\theta_h)\,\mathrm{d}(2\theta_h)\,\mathrm{d}\phi_h} = \frac{\sin\theta_h}{2\cos\theta_h \sin\theta_h \, 2} = \frac{1}{4\cos\theta_h} = \frac{1}{4(\vec{\omega}_i \cdot \vec{\omega}_h)} \; .$$

- Inserting this result gives the Torrance-Sparrow BRDF model.

Microfacet models in graphics

- Introduced by Blinn [1977].
- The Torrance-Sparrow model with different microfacet distributions (D):
  - The modified Phong [1975] model for D (cosine lobe distribution using half-vector).
  - The Torrance-Sparrow [1967] model for D (Gaussian distribution).
  - A model by Trowbridge and Reitz [1975] for D (microfacets as ellipsoids of revolution).
- There are other options as well.
  - See Cook and Torrance [1981] and Walter et al. [2007].

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The Torrance-Sparrow model

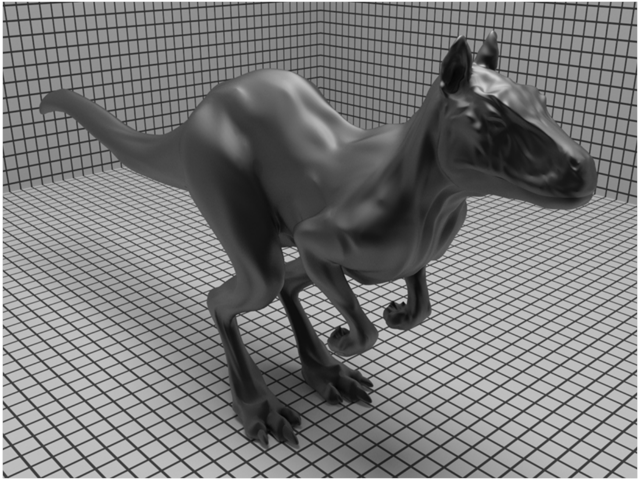

Image by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

- Using the Blinn-Phong microfacet distribution.

Newer microfacet models

- Oren-Nayar [1994].
  - Using Lambertian microfacets.
- Lafortune [1997].
  - Using multiple Phong lobes.
- Ashikhmin-Shirley [2000].
  - Two Phong lobes and Fresnel reflectance.
- Weidlich and Wilkie [2007, 2009]
  - Layered microfacet models.

References

- Oren, M., and Nayar, S. K. Generalization of Lambert's reflectance model. In *Proceedings of ACM SIGGRAPH 94*, pp. 239-246, 1994.
- Lafortune, E. P. F., Foo, S.-C., Torrance, K. E., and Greenberg, D. P. Non-linear approximation of reflectance functions. In *Proceedings of ACM SIGGRAPH 97*, pp. 117-126, 1997.
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- Weidlich, A., and Wilkie, A. Arbitrarily layered micro-facet surfaces. In *Proceedings of GRAPHITE 2007*, pp. 171–178, ACM, 2007.
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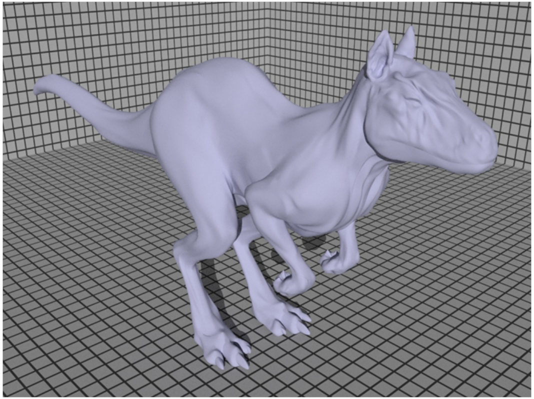
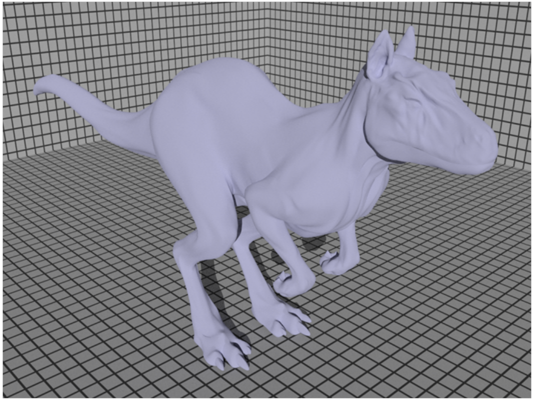
The Oren-Nayar model



Lambertian



Oren-Nayar

Images by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

The Lafortune model

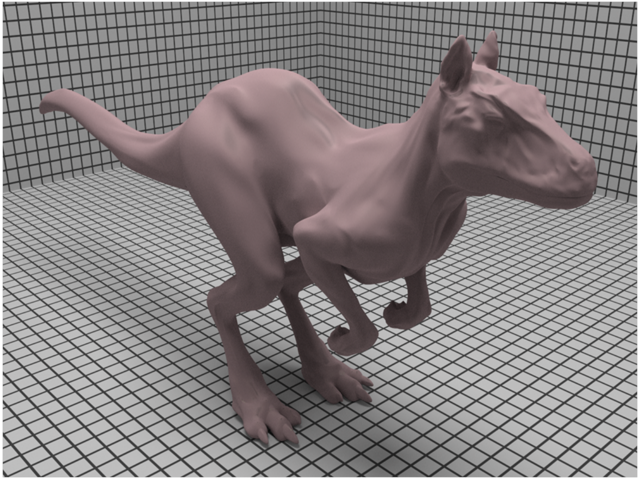


Image by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

The Ashikhmin-Shirley model

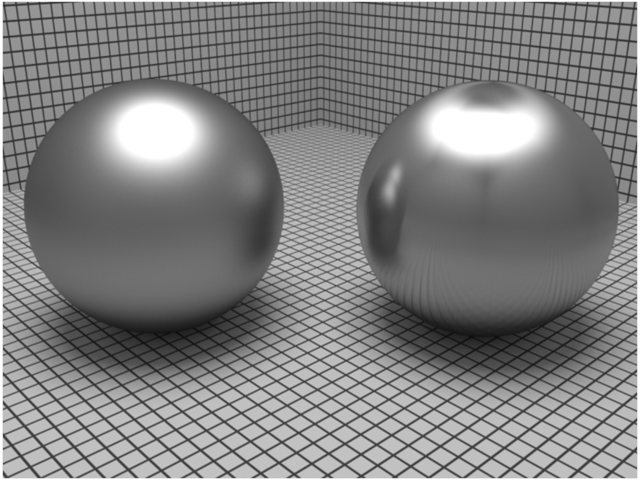


Image by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]

The Weidlich-Wilkie model

Image by Weidlich and Wilkie [2007]

Importance sampling

- The rendering equation:

$$L_o(\boldsymbol{x}, \vec{\omega}_o) = L_e(\boldsymbol{x}, \vec{\omega}_o) + \int_{2\pi} f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o) L_i(\boldsymbol{x}, \vec{\omega}_i) \cos\theta_i \, \mathrm{d}\omega_i \quad .$$

- The Monte Carlo estimator:

$$L_N(\boldsymbol{x}, \vec{\omega}_o) = L_e(\boldsymbol{x}, \vec{\omega}_o) + \frac{1}{N} \sum_{j=1}^{N} \frac{f_r(\boldsymbol{x}, \vec{\omega}_{i_j}, \vec{\omega}_o) L_i(\boldsymbol{x}, \vec{\omega}_{i_j}) \cos\theta_i}{\mathrm{pdf}(\vec{\omega}_{i_j})} \quad .$$

- **Make the pdf cancel out the BRDF or part of it.**
- The Torrance-Sparrow BRDF:

$$f_r(\boldsymbol{x}, \vec{\omega}_i, \vec{\omega}_o) = \frac{FGD}{4(\vec{n} \cdot \vec{\omega}_i)(\vec{n} \cdot \vec{\omega}_o)} = \frac{FGD}{4 \cos\theta_i \cos\theta_o} \quad .$$

- The geometry term ($\chi^+(a)$ is one if $a > 0$, zero otherwise):

$$G(\vec{\omega}_o, \vec{\omega}_i) = \chi^+\!\left(\frac{\cos\theta_h}{\cos\theta_i}\right) \chi^+\!\left(\frac{\cos\theta_h}{\cos\theta_o}\right) \min\left(1, \frac{2(\vec{n} \cdot \vec{\omega}_h)\cos\theta_o}{\cos\theta_h}, \frac{2(\vec{n} \cdot \vec{\omega}_h)\cos\theta_i}{\cos\theta_h}\right) \quad .$$

Sampling the Blinn microfacet distribution

- The Blinn microfacet normal distribution (s is shininess):

$$D(\vec{\omega}_h) = \chi^+(\vec{n}\cdot\vec{\omega}_h)\,\frac{s+2}{2\pi}(\vec{n}\cdot\vec{\omega}_h)^s \quad .$$

- Cosine lobe:

$$\mathrm{pdf}(\vec{\omega}_{i_j}) = \frac{s+2}{2\pi}\frac{1}{4\cos\theta_h}(\vec{n}\cdot\vec{\omega}_{h_j})^{s+1} \quad .$$

- Sampling technique: $\vec{\omega}_{i_j} = 2(\vec{\omega}_o\cdot\vec{\omega}_{h_j})\vec{\omega}_{h_j} - \vec{\omega}_o$ with

$$\vec{\omega}_{h_j} = (\theta,\phi) = \left(\cos^{-1}\left(\sqrt[s+2]{\xi_1}\right), 2\pi\xi_2\right) \quad ,$$

  where θ,ϕ are spherical coordinates in the tangent space of the surface normal $\vec{n}$, while $\xi_1,\xi_2 \in [0,1]$ are uniform random variables.
- The estimator

$$L_{r,N}(\boldsymbol{x},\vec{\omega}) = \frac{1}{N}\sum_{j=1}^{N} F(\vec{\omega}_{i_j}\cdot\vec{\omega}_{h_j})\,\frac{\vec{\omega}_o\cdot\vec{\omega}_{h_j}}{(\cos\theta_o)(\vec{n}\cdot\vec{\omega}_{h_j})}G(\vec{\omega}_o,\vec{\omega}_{i_j})L_i(\boldsymbol{x},\vec{\omega}_{i_j}) \quad .$$

Exercises

- Choose a microfacet normal distribution function (Blinn or Beckmann or GGX) in the paper by Walter et al. [2007].
- Implement sampling of the chosen normal distribution function.
- Implement shading of a glossy surface using a microfacet model.
- Suggested algorithm:
  - Retrieve the normal of the intersected macrosurface ($\vec{n}$).
  - Sample a microfacet normal ($\vec{m} = \vec{\omega}_h$) in the hemisphere around $\vec{n}$.
  - Perform the same operation as in shading of a transparent object, but use the sampled microfacet normal $\vec{m}$ instead of $\vec{n}$.
  - Multiply the result by the geometric attenuation factor G and the ratio of cosine terms in the estimator $\frac{\vec{\omega}_o \cdot \vec{m}}{(\vec{\omega}_o \cdot \vec{n})(\vec{n} \cdot \vec{m})}$.
- Same approach can be used for metals, but then the Fresnel factor becomes an RGB vector and everything refracted is absorbed.

References

- Walter, B., Marschner, S. R., Li, H., and Torrance, K. E. Microfacet models for refraction through rough surfaces. In *Proceedings of Eurographics Symposium on Rendering (EGSR 2007)*, pp. 195–206, 2007.