02941 Physically Based Rendering

Volume Rendering

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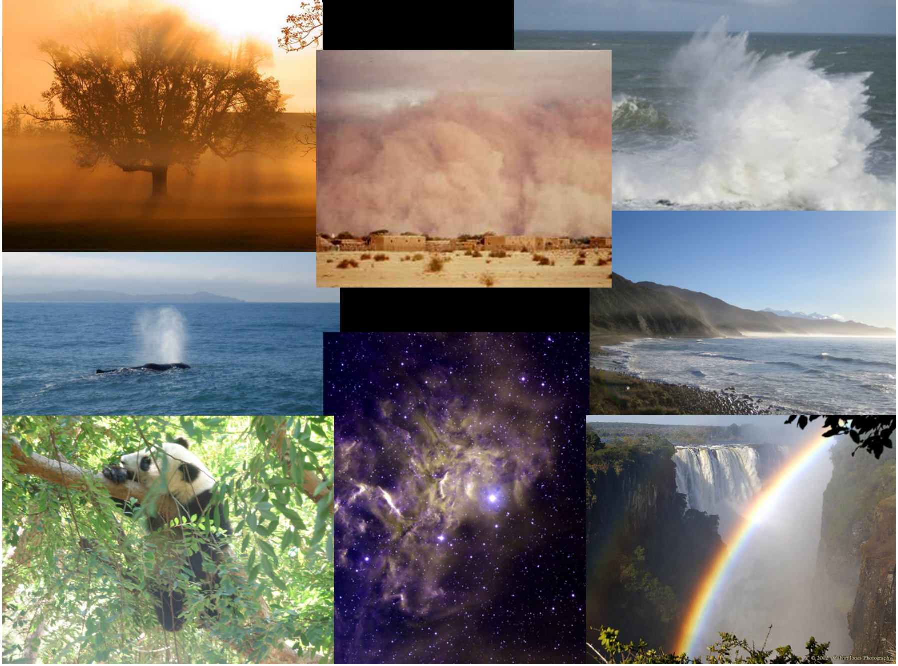
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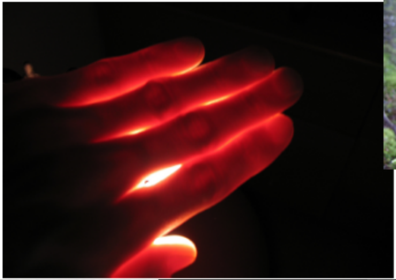





What happens in a volume?

- Some light is absorbed.
- Some light scatters away (out-scattering).
- Some light scatters back into the line of sight (in-scattering).

  (absorption + out-scattering = extinction)

- Historical origins:

Bouguer [1729, 1760]	A measure of light. Exponential extinction.
Lambert [1760]	Cosine law of perfectly diffuse reflection and emission.
Lommel [1887]	Testing Lambert's cosine law for scattering volumes.
	Describing isotropic in-scattering mathematically.
Chwolson [1889]	A theory for subsurface light diffusion (similar to Lommel's).
Schuster [1905]	Scattering in foggy atmospheres (plane-parallel media).
	Reinventing the theory in astrophysics.
King [1913]	General equation which includes anisotropic scattering (phase function).
Chandrasekhar [1950]	The first definitive text on radiative transfer.

How to describe scattering?

- We follow a ray of light passing through a scattering medium.
- The parameters describing the medium are

  σ_a the absorption coefficient $[\mathrm{m}^{-1}]$

  σ_s the scattering coefficient $[\mathrm{m}^{-1}]$

  σ_t the extinction coefficient $[\mathrm{m}^{-1}]$ ($\sigma_t = \sigma_a + \sigma_s$)

  p the phase function $[\mathrm{sr}^{-1}]$

  ε the emission properties $[\mathrm{W sr^{-1} m^{-3}}]$ (radiance per meter).
- The radiative transfer equation (RTE)

$$
\begin{aligned}
(\vec{\omega} \cdot \nabla) L(\boldsymbol{x}, \vec{\omega}) \quad = \quad & -\sigma_t(\boldsymbol{x}) L(\boldsymbol{x}, \vec{\omega}) \\
& + \sigma_s(\boldsymbol{x}) \int_{4\pi} p(\boldsymbol{x}, \vec{\omega}', \vec{\omega}) L(\boldsymbol{x}, \vec{\omega}')\, \mathrm{d}\omega' \\
& + \varepsilon(\boldsymbol{x}, \vec{\omega}) \ ,
\end{aligned}
$$

where L is radiance at the position $\boldsymbol{x}$ along the ray in the direction $\vec{\omega}$ and ε is emission.

The direct transmission term

- $(\vec{\omega} \cdot \nabla) L(\boldsymbol{x}, \vec{\omega})$ is the directional derivative along the ray.
- Absorption: $-\sigma_a(\boldsymbol{x}) L(\boldsymbol{x}, \vec{\omega})$.
- Out-scattering: $-\sigma_s(\boldsymbol{x}) L(\boldsymbol{x}, \vec{\omega})$.
- Extinction: $(-\sigma_a(\boldsymbol{x}) - \sigma_s(\boldsymbol{x})) L(\boldsymbol{x}, \vec{\omega}) = -\sigma_t(\boldsymbol{x}) L(\boldsymbol{x}, \vec{\omega})$.
- This is Bouguer's law of exponential attenuation.
- The radiative transfer equation (RTE)

$$
\begin{aligned}
(\vec{\omega} \cdot \nabla) L(\boldsymbol{x}, \vec{\omega}) \quad = \quad & \boxed{-\sigma_t(\boldsymbol{x}) L(\boldsymbol{x}, \vec{\omega})} \\
& + \sigma_s(\boldsymbol{x}) \int_{4\pi} p(\boldsymbol{x}, \vec{\omega}', \vec{\omega}) L(\boldsymbol{x}, \vec{\omega}') \, \mathrm{d}\omega' \\
& + \varepsilon(\boldsymbol{x}, \vec{\omega}) \;\; ,
\end{aligned}
$$

where L is radiance at the position $\boldsymbol{x}$ along the ray in the direction $\vec{\omega}$ and ε is emission.

The diffusion term

- $(\vec{\omega} \cdot \nabla) L(\boldsymbol{x}, \vec{\omega})$ is the directional derivative along the ray.
- In-scattering is from all directions $\vec{\omega}'$ to the ray direction $\vec{\omega}$.
- In-scattering from $\vec{\omega}'$ is weighted by the phase function p.
- In-scattering in total is weighted by the scattering coefficient σ_s.
- The radiative transfer equation (RTE)

$$\begin{aligned}(\vec{\omega} \cdot \nabla) L(\boldsymbol{x}, \vec{\omega}) &= -\sigma_t(\boldsymbol{x}) L(\boldsymbol{x}, \vec{\omega}) \\ &\quad \boxed{+ \sigma_s(\boldsymbol{x}) \int_{4\pi} p(\boldsymbol{x}, \vec{\omega}', \vec{\omega}) L(\boldsymbol{x}, \vec{\omega}') \, \mathrm{d}\omega'} \\ &\quad + \varepsilon(\boldsymbol{x}, \vec{\omega}) \ ,\end{aligned}$$

where L is radiance at the position $\boldsymbol{x}$ along the ray in the direction $\vec{\omega}$ and ε is emission.

The emission term

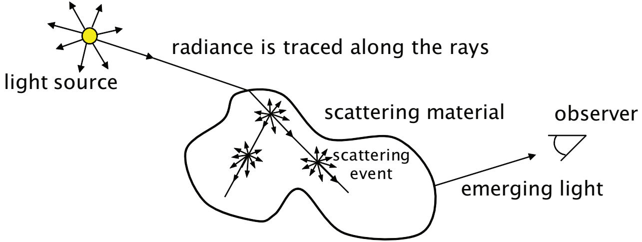
- Emission has not been investigated much in graphics.
- Volumes are typically non-emitters ($\varepsilon = 0$). Sources are usually modelled by diffusely emitting surfaces.
- It may be computed using Planck's spectrum for blackbody emission [Planck 1900, Wilkie and Weidlich 2011].
- The radiative transfer equation (RTE)

$$\begin{aligned}(\vec{\omega} \cdot \nabla)L(\boldsymbol{x}, \vec{\omega}) &= -\sigma_t(\boldsymbol{x})L(\boldsymbol{x}, \vec{\omega}) \\ &\quad + \sigma_s(\boldsymbol{x}) \int_{4\pi} p(\boldsymbol{x}, \vec{\omega}', \vec{\omega})L(\boldsymbol{x}, \vec{\omega}')\, \mathrm{d}\omega' \\ &\quad \boxed{+\,\varepsilon(\boldsymbol{x}, \vec{\omega})}\ ,\end{aligned}$$

where L is radiance at the position $\boldsymbol{x}$ along the ray in the direction $\vec{\omega}$ and ε is emission.

Rendering volumes

- The general method: path tracing (Monte Carlo integration).



- The integral form of the radiative transfer equation (for a non-emitter):

$$L(s) = T_r(0, s)L(0) + \int_0^s T_r(s', s)\sigma_s(s') \int_{4\pi} p(s', \vec{\omega}', \vec{\omega})L(s', \vec{\omega}')\, \mathrm{d}\omega'\, \mathrm{d}s' \,,$$

where T_r is the beam transmittance and s is the distance travelled along a ray with direction $\vec{\omega}$ and origin $\boldsymbol{o}$ on the surface of the volume such that $\boldsymbol{x} = \boldsymbol{o} + s\,\vec{\omega}$ is a point along the ray inside the volume.

Direct transmission

- Direct transmission is the first term of the RTE:

$$L_{\text{transmission}}(s) = T_r(0, s)L(0) \quad .$$

- Beam transmittance: $T_r(s', s) = e^{-\tau(s', s)}$.
- Optical thickness: $\tau(s', s) = \int_{s'}^{s} \sigma_t(t)\, \mathrm{d}t$.
- Ray points:

  $s' = 0$ point of incidence.
  $s' = s$ point inside or point of emergence.

- For homogeneous materials: $T_r(s', s) = e^{-\sigma_t(s - s')}$.
- Then

$$L_{\text{transmission}}(s) = e^{-\sigma_t s} L(0) \quad ,$$

  where σ_t is the extinction coefficient, s is the distance to the surface, and $L(0)$ is the radiance refracted into the medium at the point of incidence.

Diffusion (in-scattering)

- Diffusion is the second term of the RTE:

$$L_{\text{diffusion}}(s) = \int_0^s T_r(s', s)\sigma_s(s')J(s')\,\mathrm{d}s' \quad ,$$

  where J is the source function:

$$J(s') = \int_{4\pi} p(s', \vec{\omega}', \vec{\omega})L(s', \vec{\omega}')\,\mathrm{d}\omega' \quad .$$

- Monte Carlo estimator for the diffusion term:

$$L_{\text{diffusion},N} = \frac{1}{N}\sum_{j=1}^{N} \frac{T_r(s'_j, s)\sigma_s(s'_j)J(s'_j)}{\text{pdf}(s'_j)} \quad .$$

- We know how to sample an exponential function (see slides on MC integration)

$$\text{pdf}(s'_j) = \sigma_t(s'_j)\,T_r(s'_j, s) = \sigma_t e^{-\sigma_t(s-s'_j)} \quad , \quad s - s'_j = -\frac{\ln(1-\xi_j)}{\sigma_t} \quad .$$

Distance to next scattering event

- Monte Carlo estimator for the diffusion term:

$$L_{\text{diffusion},N} = \frac{1}{N}\sum_{j=1}^{N} \frac{T_r(s'_j, s)\sigma_s(s'_j)J(s'_j)}{\text{pdf}(s'_j)} \quad .$$

- We know how to sample an exponential function (see slides on MC integration)

$$\text{pdf}(s'_j) = \sigma_t(s'_j)\,T_r(s'_j, s) = \sigma_t e^{-\sigma_t(s-s'_j)} \quad , \quad s - s'_j = -\frac{\ln(1-\xi_j)}{\sigma_t} \quad .$$

- We are interested in the radiance $L(s)$. Then $d_j = s - s'_j$ is the sampled distance to the next scattering event.
- If d_j is greater than the distance to the surface s, the next scattering event is refraction through the surface.
- This refraction accounts for the direct transmission term (since it corresponds to a Russian roulette using T_r to decide if the next event is scattering or direct transmission).

The scattering albedo

- Monte Carlo estimator for the diffusion term:

$$L_{\text{diffusion},N} = \frac{1}{N}\sum_{j=1}^{N} \frac{T_r(s'_j, s)\sigma_s(s'_j)J(s'_j)}{\text{pdf}(s'_j)} \quad .$$

- Inserting the pdf, we have

$$L_{\text{diffusion},N} = \frac{1}{N}\sum_{j=1}^{N} \frac{\sigma_s(s'_j)}{\sigma_t(s'_j)} J(s'_j) \quad .$$

- The scattering albedo: $\alpha = \sigma_s/\sigma_t$.
- Using Russian roulette with the scattering albedo:

$$L_{\text{diffusion},N} = \begin{cases} \frac{1}{N}\sum_{j=1}^{N} J(s'_j) & \text{for } \xi < \alpha \\ 0 & \text{otherwise} \end{cases}$$

The source function

- The source function is in-scattering from all directions:

$$J(s') = \int_{4\pi} p(s', \vec{\omega}', \vec{\omega}) L(s', \vec{\omega}') \, \mathrm{d}\omega' \ .$$

- Monte Carlo estimator for the source function:

$$J_N = \frac{1}{M} \sum_{k=1}^{M} \frac{p(s', \vec{\omega}'_k, \vec{\omega}) L(s', \vec{\omega}'_k)}{\mathrm{pdf}(\vec{\omega}'_k)} \ .$$

- Importance sampling: Use a pdf similar to the phase function.
- For isotropic media: $p = \frac{1}{4\pi}$. Sample the unit sphere uniformly.
- Anisotropic media with rotationally invariant scattering $(p(\vec{\omega}', \vec{\omega}) = p(\vec{\omega}' \cdot \vec{\omega}))$ are described by the asymmetry parameter:

$$g = \int_{4\pi} p(\vec{\omega}' \cdot \vec{\omega})(\vec{\omega}' \cdot \vec{\omega}) \, \mathrm{d}\omega' \ ,$$

which is the mean cosine of the scattering angle.

The Henyey-Greenstein phase function

- Henyey and Greenstein [1940] suggested a phase function based on the asymmetry parameter g:

$$p(\vec{\omega}' \cdot \vec{\omega} = \cos\theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g\cos\theta)^{3/2}} \quad .$$

- The HG phase function follows the properties of g

  $g = -1$ total backscattering
  $g = 0$ isotropic scattering
  $g = 1$ total forward scattering.

- It is also a spherical harmonics expansion of the phase function with coefficients $c_n = g^n$.
- There is a simple way to importance sample it [Hanrahan and Krueger 1992, Pharr and Humphreys 2004; 2010; 2017]:

$$\cos\theta_k = \begin{cases} \frac{1}{2g}\left[1 + g^2 - \left(\frac{1-g^2}{1-g+2g\xi_k}\right)^2\right] & \text{for } g \neq 0 \\ 2\xi_k - 1 & \text{for } g = 0 \end{cases} \quad .$$

Path tracing volumes

- In path tracing, we usually take only one sample for each estimator per frame ($N = M = 1$).
- When a ray hits a scattering material, do the following (j is iteration number).
  1. If the ray hit from outside, do a standard volume transmission and stop.
     If the ray hit from the inside, proceed.
  2. Sample the distance $d_1 = -\ln(1 - \xi_1)/\sigma_t$ to the next scattering event. If the scattering event is outside the volume ($d_1 > s$), do a transparent object transmission and stop. Otherwise, proceed.
  3. Do a Russian roulette with the scattering albedo. If $\xi_{2j} > \alpha$, the ray is absorbed. Otherwise, proceed.
  4. Sample the distance to the next scattering event $d_2 = -\ln(1 - \xi_{2j+1})/\sigma_t$.
  5. Create a scatter ray and set its maximum trace distance ($t_{\max}$) to d_2.
  6. Trace the scatter ray from the origin $\boldsymbol{o}_2 = \boldsymbol{o}_1 + d_1\,\vec{\omega}_1$ in a direction $\vec{\omega}_2$ obtained by sampling the phase function. If it does not hit something, copy d_2 to d_1, let the scatter ray overwrite the old ray, and proceed to step 3.
     Otherwise, do a transparent object transmission and stop.
- This procedure only works for monochromatic rays.

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