# 02941 Physically Based Rendering

## Dispersion and Spectral Rendering

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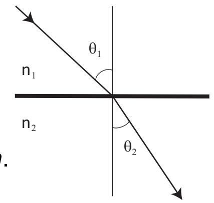
## Dispersion

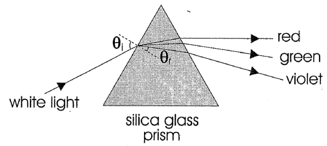
- The law of refraction:




$$n_1(\lambda)\sin\theta_1 = n_2(\lambda)\sin\theta_2 \ .$$

where $\lambda$ is the wavelength *in vacuum*.

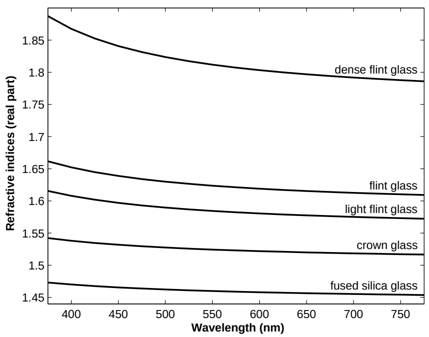
- Light of different wavelengths is refracted differently.
- The classic example: dispersion by a glass prism.




(Fused silica glass is, however, not very dispersive.)

## Refractive indices of glasses

- Glasses of larger density exhibit stronger dispersion.




- What type of glass would you use for a dispersion prism?

## The index of refraction

- The index of refraction is a complex number defined (for isotropic materials, see slides on reflection and transmission) by

$$n = n' + in'' = c\sqrt{\mu(\varepsilon + i\sigma/\omega)} \ ,$$

where
  - $\omega = 2\pi c/\lambda$ is the angular frequency and $c$ is the speed of light *in vacuum*,
  - $\varepsilon$ is the permittivity, $\mu$ is the permeability, and $\sigma$ is the conductivity of the material.
- The square root of a complex number is

$$\sqrt{a + ib} = \sqrt{\frac{\sqrt{a^2 + b^2} + a}{2}} + i\,\mathrm{sgn}(b)\sqrt{\frac{\sqrt{a^2 + b^2} - a}{2}} \ .$$

- This means that conductivity leads to absorption, but it also changes the real part of the refractive index.

## Absorption and dispersion

- Using the formula for the square root of a complex number, the index of refraction becomes

$$n = c\sqrt{\frac{\mu}{2}}\left(\sqrt{\sqrt{\varepsilon^2 + \frac{\sigma^2\lambda^2}{4\pi^2c^2}} + \varepsilon} + i\sqrt{\sqrt{\varepsilon^2 + \frac{\sigma^2\lambda^2}{4\pi^2c^2}} - \varepsilon}\right) .$$

- Thus there is an internal relation between $n'$ and $n''$ (Kramers-Kronig relations).
- Only some combinations of $n'$ and $n''$ can occur in nature.
- Absorption affects the phase velocity ($c/n'$) of light in a medium and vice versa.
- In the visible part of the spectrum, $n'$ normally decreases with increasing wavelength (as for transparent glass).
- Larger absorption ($n''$) $\leftrightarrow$ larger conductivity ($\sigma$) $\rightarrow n'$ may increase with increasing wavelength ($\lambda$).
- This case is called *anomalous dispersion*.

## Rendering dispersion

- Problem: We usually trace one ray for the combined RGB representation of the colour, not one per wavelength.
- Simple solution: Split rays that intersect dispersive objects in three, one ray for each colour band (R, G, and B).
  - Use Russian roulette to avoid a combinatorial explosion.
  - Each colour band is of equal importance (pdf $= 1/3$).
- Better solution: Do spectral sampling. Sample a wavelength using Russian roulette and trace a ray for this wavelength.
- New problems:
  - How do we sample?
  - Are some wavelengths in the visible part of the spectrum more important than others?
  - How do we best convert back to RGB?
- Solution: Use the CIE RGB colour matching functions. But use them in the right way ...

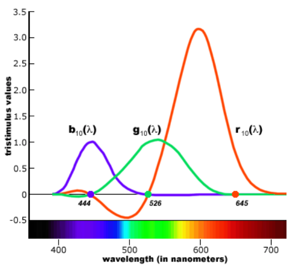
## Spectral rendering

- Transformation between spectrum and RGB colour values:

$$R = \int_{\mathscr{V}} C(\lambda)\bar{r}(\lambda)\,\mathrm{d}\lambda\,,\quad G = \int_{\mathscr{V}} C(\lambda)\bar{g}(\lambda)\,\mathrm{d}\lambda\,,\quad B = \int_{\mathscr{V}} C(\lambda)\bar{b}(\lambda)\,\mathrm{d}\lambda\ ,$$

  where $\mathscr{V}$ denotes the interval of visible wavelengths (approximately from 380 nm to 780 nm) and $C(\lambda)$ is the spectrum that we want to transform to RGB.

- $\bar{r}$, $\bar{g}$, and $\bar{b}$ are normalized 10° RGB colour matching functions of Stiles and Burche [1959].
- To sample these integrals, we sample a wavelength using a step function (Russian roulette).




## Spectral sampling using the colour matching functions

- The goal is to sample an index $i$ into a tabulated spectrum of refractive indices of a dispersive material.
- The RGB colour matching functions are available for $\lambda \in [390\,\text{nm}, 830\,\text{nm}]$ with a spectral resolution of 5 nm.
- We can use the mean of $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ as the importance of the wavelengths in $[\lambda, \lambda + 5\,\text{nm}]$.
- To sample this, we test a uniform random variable $\xi \in [0,1]$ against the cumulative distribution function (cdf): The index $j$ of the largest cdf element smaller than $\xi$ is found using binary search.
- The wavelength $\lambda$ corresponding to a sampled index $j$ is then

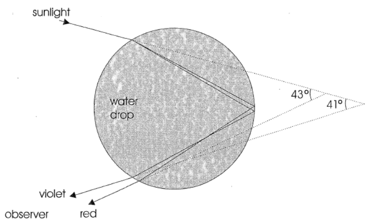
$$\lambda = \lambda_{\text{rgb,min}} + j\Delta\lambda_{\text{rgb}}$$

  while the sampled index into the table of refractive indices is

$$i = \left\lfloor \frac{\lambda - \lambda_{ior,\text{min}}}{\Delta\lambda_{\text{ior}}} \right\rfloor .$$

# Rainbows (particle scattering teaser)

- Rainbows are often attributed to dispersion.



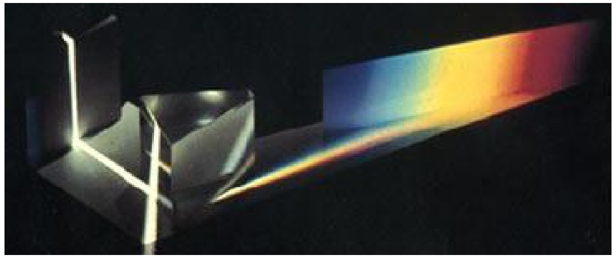

- However, dispersion is not the whole story [Sadeghi et al. 2012].
- A rainbow is more precisely referred to as a particle scattering phenomenon.
- Volume rendering is necessary here. This topic is covered in later worksheets.

References

- Sadeghi, I., Muñoz, A., Laven, P., Jarosz, W., Seron, F., Gutierrez, D., and Jensen, H. W. Physically-based simulation of rainbows. *ACM Transactions on Graphics 31*(1), pp. 3:1–3:12, January 2012.

# Exercises

- Render an image qualitatively comparable to the following photograph by Williamson and Cummins [1983], also in the paper by Sun et al. [2000].



- Explain why the photon mapping result differs from the photo.
- Explain why the prism in the photo is probably made of dense flint glass.