02941 Physically Based Rendering

Dispersion and Spectral Rendering

Jeppe Revall Frisvad

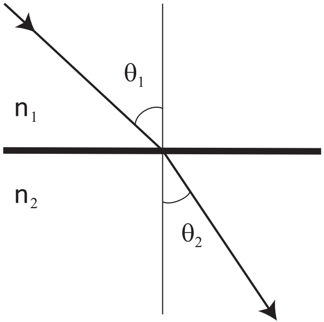
June 2022

Dispersion

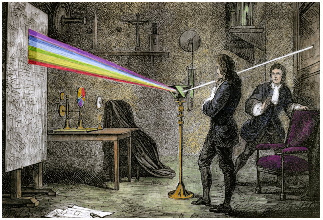
- The law of refraction:

$$n_1(\lambda)\sin\theta_1 = n_2(\lambda)\sin\theta_2 \quad .$$

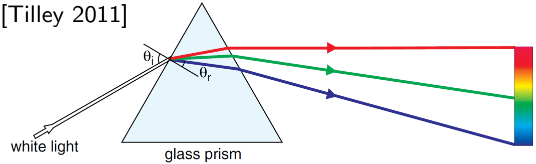
where λ is the wavelength *in vacuo*.



- Light of different wavelengths is refracted differently.
- The classic example: dispersion by a glass prism.



[Tilley 2011]

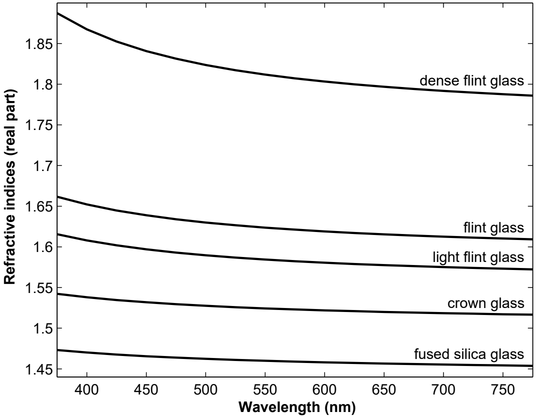


References

- Tilley, R. Colour due to refraction and dispersion. In *Colour and the Optical Properties of Materials*, second edition, Chapter 2, pp. 49-90. John Wiley & Sons, 2011.

Refractive indices of glasses

- Glasses of larger density exhibit stronger dispersion (data from Tropf et al. [1995]).



- What type of glass would you use for a dispersion prism?

References

- Tropf, W. J., Thomas, M. E., Harris, T. J. Properties of crystals and glasses. In *Handbook of Optics: Devices, Measurements, and Properties*, Vol. 2, second edition. McGraw-Hill, 1995.

The index of refraction

- The index of refraction is a complex number defined (for isotropic materials, see slides on reflection and transmission) by

$$n = n' + in'' = c\sqrt{\mu(\varepsilon + i\sigma/\omega)} \ ,$$

  where
  - $\omega = 2\pi c/\lambda$ is the angular frequency and c is the speed of light *in vacuo*,
  - ε is the permittivity, μ is the permeability, and σ is the conductivity of the material.
- The square root of a complex number is

$$\sqrt{a + ib} = \sqrt{\frac{\sqrt{a^2 + b^2} + a}{2}} + i\,\mathrm{sgn}(b)\sqrt{\frac{\sqrt{a^2 + b^2} - a}{2}} \ .$$

- This means that conductivity leads to absorption, but it also changes the real part of the refractive index.

Absorption and dispersion

- Using the formula for the square root of a complex number, the index of refraction becomes

$$n = c\sqrt{\frac{\mu}{2}}\left(\sqrt{\sqrt{\varepsilon^2 + \frac{\sigma^2\lambda^2}{4\pi^2c^2}} + \varepsilon} + i\sqrt{\sqrt{\varepsilon^2 + \frac{\sigma^2\lambda^2}{4\pi^2c^2}} - \varepsilon}\right) .$$

- Thus there is an internal relation between n' and n'' (Kramers-Kronig relations).
- Only some combinations of n' and n'' can occur in nature.
- Absorption affects the phase velocity (c/n') of light in a medium and vice versa.
- In the visible part of the spectrum, n' normally decreases with increasing wavelength (as for transparent glass).
- Larger absorption (n'') $\leftrightarrow$ larger conductivity (σ)
  $\rightarrow$ n' may increase with increasing wavelength (λ).
- This case is called *anomalous dispersion*.

Rendering dispersion

- Problem: We usually trace one ray for the combined RGB representation of the colour, not one per wavelength.
- Simple solution: Split rays that intersect dispersive objects in three, one ray for each colour band (R, G, and B).
  - Use Russian roulette to avoid a combinatorial explosion.
  - Each colour band is of equal importance (pdf $= 1/3$).
- Better solution: Do spectral sampling. Sample a wavelength using Russian roulette and trace a ray for this wavelength.
- New problems:
  - How do we sample?
  - Are some wavelengths in the visible part of the spectrum more important than others?
  - How do we best convert back to RGB?
- Solution: Use the CIE RGB colour matching functions. But use them in the right way ...

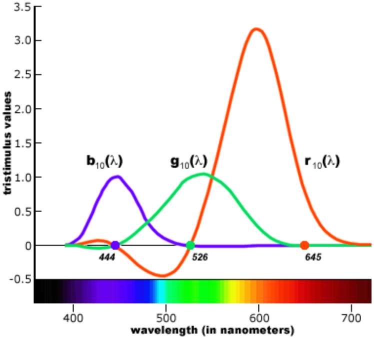
Spectral rendering

- Transformation between spectrum and RGB colour values:

$$R = \int_{\mathscr{V}} C(\lambda)\bar{r}(\lambda)\,\mathrm{d}\lambda\,, \quad G = \int_{\mathscr{V}} C(\lambda)\bar{g}(\lambda)\,\mathrm{d}\lambda\,, \quad B = \int_{\mathscr{V}} C(\lambda)\bar{b}(\lambda)\,\mathrm{d}\lambda\,,$$

where $\mathscr{V}$ denotes the interval of visible wavelengths (approximately from 380 nm to 780 nm) and $C(\lambda)$ is the spectrum that we want to transform to RGB.

- $\bar{r}$, $\bar{g}$, and $\bar{b}$ are 10° RGB colour matching functions of Stiles and Burche [1959].
- To sample these integrals, we sample a wavelength using a discrete pdf (similar to Russian roulette).



- Stiles, W. S., and Burche, J. M. N.P.L. colour-matching investigation: Final report (1958). *Optica Acta 6*, 1–26. 1959.

Sampling of a discrete pdf (tabulation)

- Suppose we have n samples x_i, $i = 1, \dots, n$, uniformly distributed with intervals of length Δx across the domain of the pdf.
- With $\mathrm{cdf}(x_0) = 0$, the ith element of the tabulated cdf has

$$\mathrm{cdf}(x_i) = \sum_{j=1}^{i} \mathrm{pdf}(x_j)\Delta x = \mathrm{pdf}(x_i)\Delta x + \mathrm{cdf}(x_{i-1})\,.$$

- We then draw a random number $\xi \in [0, 1)$ and use binary search to find j such that $\mathrm{cdf}(x_{j-1}) \leq \xi < \mathrm{cdf}(x_j)$.
- The sampled value is then

$$x = x_{\min} + \Delta x \left(j - \frac{\mathrm{cdf}(x_j) - \xi}{\mathrm{cdf}(x_j) - \mathrm{cdf}(x_{j-1})} \right)\,.$$

- The pdf of the sampled value is

$$\mathrm{pdf}(x) = \mathrm{cdf}(x_j) - \mathrm{cdf}(x_{j-1})\,.$$

Spectral sampling using the colour matching functions

- The goal is to sample an index i into a tabulated spectrum of refractive indices of a dispersive material.
- The RGB colour matching functions are available for $\lambda \in [390\,\text{nm}, 830\,\text{nm}]$ with a spectral resolution of $5\,\text{nm}$.
- We can use the element-by-element mean of **normalized** $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ as a discrete pdf for sampling of a wavelength λ (see previous slide).
- The sampled index into the table of refractive indices is

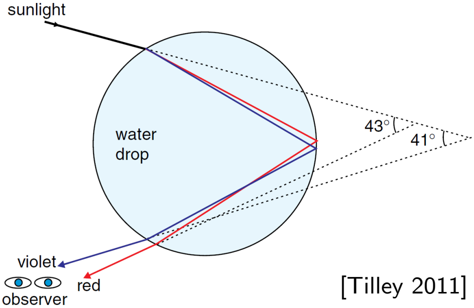
$$i = \left\lfloor \frac{\lambda - \lambda_{\text{ior,min}}}{\Delta\lambda_{\text{ior}}} \right\rfloor \quad .$$

- Linear interpolation can be used in the look-up of the refractive index (ior).
- The RGB flux (or radiance) carried along a ray with a sampled wavelength λ should be weighted element-wise by the colour matching functions:

$$(\Phi_r, \Phi_g, \Phi_b) = \Phi_\lambda \frac{(\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda))}{\text{pdf}(\lambda)} \,.$$

Rainbows (particle scattering teaser)

- Rainbows are often attributed to dispersion.



[Tilley 2011]

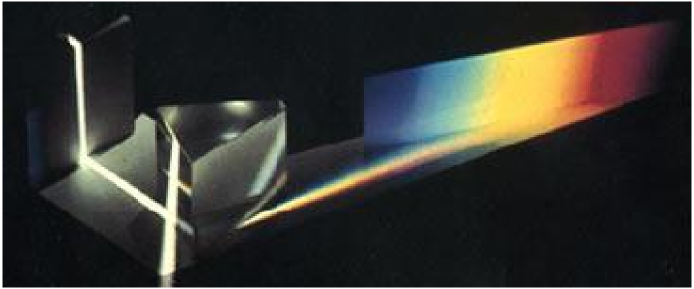
- However, dispersion is not the whole story [Sadeghi et al. 2012].
- A rainbow is more precisely referred to as a particle scattering phenomenon.
- Volume rendering is necessary here. This topic is covered in another lecture.

References

- Sadeghi, I., Muñoz, A., Laven, P., Jarosz, W., Seron, F., Gutierrez, D., and Jensen, H. W. Physically-based simulation of rainbows. *ACM Transactions on Graphics 31*(1), pp. 3:1–3:12, January 2012.

Exercises

- Render an image qualitatively comparable to the following photograph by Williamson and Cummins [1983], also in the paper by Sun et al. [2000].



- Explain why the photon mapping result differs from the photo.
- Explain why the prism in the photo is probably made of dense flint glass.

References

- Williamson, S. J., and Cummins H. Z. *Light and Color in Nature and Art*. John Wiley & Sons, 1983.
- Sun, Y., Fracchia, F. D., and Drew, M. S. Rendering light dispersion with a composite spectral model. In *Proceedings of Color in Graphics and Image Processing (CGIP 2000)*, pp. 51–56. 2000.