02941 Physically Based Rendering

Microfacet Models

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From smooth to rough surfaces

- Triangles can represent any surface.
  - Smooth: Use interpolated vertex normals.
  - Rough: Use triangle face normals.

- If surface features are very small:
  - Triangles are very small (numerical problems).
  - Triangles are numerous (computation is expensive).

- Microfacet BRDF models.
  - A BRDF can replace tiny surface features with randomly oriented microfacets.
In general, we have the bidirectional scattering-surface reflectance distribution function (BSSRDF):

\[ S(X; x_i, \omega_i; x_o, \omega_o) = \frac{dL_o(x_o, \omega_o)}{d\Phi_i(x_i, \omega_i)}, \]

where \( X \) is the object boundary and \( L_o \) is the radiance reflected in the direction \( \omega_o \) at the position \( x_o \) due to the flux incident at the position \( x_i \) from the direction \( \omega_i \).

Suppose we consider surface scattering only, or interior scattering so large that an element of irradiance \( (dE = L_i \cos \theta_i d\omega_i) \) has an influence only for \( x_o \in A_i \), then

\[
\int_{A_i} S(X; x_i, \omega_i; x_o, \omega_o) \, dA_i \rightarrow \frac{dL_o(x_o, \omega_o)}{d\Phi_i(x_i, \omega_i)} \, dA = \frac{dL_o(x, \omega_o)}{dE(x, \omega_i)} = f_r(x, \omega_i, \omega_o)
\]

for \( A_i \rightarrow dA_o \) such that \( x_i \approx x_o = x \).

This is a special case of the BRDF \( (f_r) \) for which we may assume \( dA_i = dA_o = dA \).
Microfacet surfaces

- A microfacet surface is modelled by a BRDF that scatters light in more than one direction.
- One way to describe this is by a distribution of microfacet normals.

Figure by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]
The origins

- The scattering of electromagnetic waves from rough surfaces.
  - Overview by Beckmann and Spizzichino [1963].
  - How to develop facet normal distribution functions.

- Translation to geometrical optics.
  - Theory for off-specular reflection from roughened surfaces by Torrance and Sparrow [1967].
  - Introducing the BRDF model

\[ f_r(x, \vec{w}_i, \vec{w}_o) = \frac{FGD}{4(\vec{n} \cdot \vec{w}_i)(\vec{n} \cdot \vec{w}_o)} = \frac{FGD}{4 \cos \theta_i \cos \theta_o}, \]

where
- \( F \) is the Fresnel reflectance,
- \( G \) is the geometrical attenuation factor,
- \( D \) is the facet normal distribution function.

References

Geometrical attenuation

Important effects to consider:

(a) Masking.
(b) Shadowing.
(c) Interreflections.

Figure by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]
Facet normal distributions

- Assuming perfectly specular microfacets, facets with normals
  \[
  \vec{\omega}_h = \frac{\vec{\omega}_o + \vec{\omega}_i}{\|\vec{\omega}_o + \vec{\omega}_i\|}
  \]
  contribute to the reflected radiance in the direction \(\vec{\omega}_o\).

- Subscripts denote: \(i\) - in, \(o\) - out, \(h\) - half.

- \(D(\vec{\omega}_h)\) is the facet normal distribution. Then
  - \(D(\vec{\omega}_h) d\omega_h\) is the proportion of facets with normals in the solid angle \(d\omega_h\).
  - \(D(\vec{\omega}_h) d\omega_h dA\) is the proportion of facets in the area \(dA\) with normals in \(d\omega_h\).
  - \(D(\vec{\omega}_h) d\omega_h \cos \theta_h dA\) is the proportion of facets in the projected area \(\cos \theta_h dA\) with normals in \(d\omega_h\), where \(\cos \theta_h = \vec{\omega}_i \cdot \vec{\omega}_h\).

- The flux incident at the projected area of the microfacets with normals in \(d\omega_h\) is therefore (using the definition of radiance)
  \[
  d\Phi_i = L_i D(\vec{\omega}_h) d\omega_h \cos \theta_h dA \ d\omega_i ,
  \]
  where \(L_i\) is radiance incident from the direction \(\vec{\omega}_i\).
Facet normal distributions

- The flux incident at the projected area of the microfacets with normals in $d\omega_h$ is therefore (using the definition of radiance)

$$d\Phi_i = L_i D(\vec{\omega}_h) \, d\omega_h \, \cos \theta_h \, dA \, d\omega_i,$$

where $L_i$ is radiance incident from the direction $\vec{\omega}_i$.

- Taking into account Fresnel reflectance and geometrical attenuation, the reflected flux is

$$d\Phi_o = F(\cos \theta_h) G(\vec{\omega}_i, \vec{\omega}_o) \, d\Phi_i.$$

- The BRDF model is then (using $dE = L_i \cos \theta_i \, d\omega_i$)

$$f_r(x, \vec{\omega}_i, \vec{\omega}_o) = \frac{dL_o}{dE} = \frac{d\Phi_o}{\cos \theta_o \, dA \, d\omega_o} / dE = F(\cos \theta_h) G(\vec{\omega}_i, \vec{\omega}_o) L_i D(\vec{\omega}_h) \, d\omega_h \, \cos \theta_h \, dA \, d\omega_i \over \cos \theta_o \, dA \, d\omega_o \, L_i \, \cos \theta_i \, d\omega_i.$$
Facet normal distributions

- The BRDF model is then

\[
f_r(\mathbf{x}, \bar{\omega}_i, \bar{\omega}_o) = \frac{d\omega_h \cos \theta_h F(\cos \theta_h) G(\bar{\omega}_i, \bar{\omega}_o) D(\bar{\omega}_h)}{d\omega_o \cos \theta_i \cos \theta_o}.
\]

- Expressing the solid angles in spherical coordinates with \(\bar{\omega}_i\) as zenith, we have

\[
d\omega_h = \sin \theta_h d\theta_h d\phi_h,
\]

\[
d\omega_o = \sin \theta'_o d\theta'_o d\phi'_o.
\]

where \(\theta'_o\) is the angle between \(\bar{\omega}_i\) and \(\bar{\omega}_o\).
- Then according to the law of reflection \(\phi'_o = \phi_h\) and \(\theta'_o = 2\theta_h\).
- This means that

\[
\frac{d\omega_h}{d\omega_o} = \frac{\sin \theta_h d\theta_h d\phi_h}{\sin(2\theta_h) d(2\theta_h) d\phi_h} = \frac{\sin \theta_h}{2 \cos \theta_h \sin \theta_h 2} = \frac{1}{4 \cos \theta_h} = \frac{1}{4(\bar{\omega}_i \cdot \bar{\omega}_h)}.
\]
- Inserting this result gives the Torrance-Sparrow BRDF model.
Microfacet models in graphics

- Introduced by Blinn [1977].
- The Torrance-Sparrow model with different microfacet distributions (D):
  - The Torrance-Sparrow [1967] model for D (Gaussian distribution).
- There are other options as well.
  - See Cook and Torrance [1981] and Walter et al. [2007].

References

The Torrance-Sparrow model

Using the Blinn-Phong microfacet distribution.
Newer microfacet models

- Oren-Nayar [1994].
  - Using Lambertian microfacets.

- Lafortune [1997].
  - Using multiple Phong lobes.

- Ashikhmin-Shirley [2000].
  - Two Phong lobes and Fresnel reflectance.

- Weidlich and Wilkie [2007, 2009]
  - Layered microfacet models.

References

The Oren-Nayar model

Lambertian

Oren-Nayar

Images by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]
The Lafortune model

Image by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]
The Ashikhmin-Shirley model

Image by Pharr and Humphreys [Physically Based Rendering, Morgan Kaufmann/Elsevier, 2004]
The Weidlich-Wilkie model

Image by Weidlich and Wilkie [2007]
Importance sampling

▶ The rendering equation:
\[
L_o(x, \vec{\omega}_o) = L_e(x, \vec{\omega}_o) + \int_{2\pi} f_r(x, \vec{\omega}_i, \vec{\omega}_o) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\omega_i.
\]

▶ The Monte Carlo estimator:
\[
L_N(x, \vec{\omega}_o) = L_e(x, \vec{\omega}_o) + \frac{1}{N} \sum_{j=1}^{N} \frac{f_r(x, \vec{\omega}_{ij}, \vec{\omega}_o) L_i(x, \vec{\omega}_{ij}) \cos \theta_i}{\text{pdf}(\vec{\omega}_{ij})}.
\]

▶ Make the pdf cancel out the BRDF or part of it.
▶ The Torrance-Sparrow BRDF:
\[
f_r(x, \vec{\omega}_i, \vec{\omega}_o) = \frac{FGD}{4(\vec{n} \cdot \vec{\omega}_i)(\vec{n} \cdot \vec{\omega}_o)} = \frac{FGD}{4 \cos \theta_i \cos \theta_o}.
\]

▶ The geometry term:
\[
G(\vec{\omega}_o, \vec{\omega}_i) = \min \left(1, \frac{2(\vec{n} \cdot \vec{\omega}_h) \cos \theta_o}{\vec{\omega}_i \cdot \vec{\omega}_h}, \frac{2(\vec{n} \cdot \vec{\omega}_h) \cos \theta_i}{\vec{\omega}_i \cdot \vec{\omega}_h} \right).
\]
Sampling the Blinn microfacet distribution

- The Blinn microfacet normal distribution ($s$ is shininess):

\[ D(\bar{\omega}_h) = \frac{s+2}{2\pi}(\bar{n} \cdot \bar{\omega}_h)^s. \]

- Cosine lobe:

\[ \text{pdf}(\bar{\omega}_{ij}) = \frac{s+2}{2\pi} \frac{1}{4 \cos \theta_h} (\bar{n} \cdot \bar{\omega}_{hj})^{s+1}. \]

- Sampling technique: \( \bar{\omega}_{ij} = 2(\bar{\omega}_o \cdot \bar{\omega}_{hj})\bar{\omega}_{hj} - \bar{\omega}_o \) with

\[ \bar{\omega}_{hj} = (\theta_h, \phi) = \left( \cos^{-1}\left(\frac{s+2}{\sqrt{\xi_1}}\right), 2\pi \xi_2 \right), \]

where \( \xi_1, \xi_2 \in [0, 1] \) are uniform random variables.

- The estimator

\[ L_{r,N}(x, \bar{\omega}) = \frac{1}{N} \sum_{j=1}^{N} F(\bar{\omega}_{ij} \cdot \bar{\omega}_{hj}) \frac{|\bar{\omega}_o \cdot \bar{\omega}_{hj}|}{|\cos \theta_o||\bar{n} \cdot \bar{\omega}_{hj}|} G(\bar{\omega}_o, \bar{\omega}_i)L_i(x, \bar{\omega}_{ij}). \]
Exercises

▶ Choose a microfacet normal distribution function (Blinn or Beckmann or GGX) in the paper by Walter et al. [2007].
▶ Implement sampling of the chosen normal distribution function.
▶ Implement shading of a glossy surface using a microfacet model.
▶ Suggested algorithm:
  - Retrieve the normal of the intersected macrosurface ($\vec{n}$).
  - Sample a microfacet normal ($\vec{m} = \vec{\omega}_h$) in the hemisphere around $\vec{n}$.
  - Perform the same operation as in shading of a transparent object, but use the sampled microfacet normal $\vec{m}$ instead of $\vec{n}$.
  - Multiply the result by the geometric attenuation factor $G$ and the ratio of cosine terms in the estimator $\frac{|\vec{\omega}_o \cdot \vec{m}|}{|\vec{\omega}_o \cdot \vec{n}| |\vec{n} \cdot \vec{m}|}$.
▶ Same approach can be used for metals, but then the Fresnel factor becomes an RGB vector and everything refracted is absorbed.

References